QUASIDISSIPATIVE SYSTEMS WITH ONE OR SEVERAL SUPPLY RATES

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Abstract

The new versions of dissipativity-like concepts are introduced. It is shown that sequential quasidissipativity for several supply rates is equivalent to that with one supply rate which is their convex combination. The key point of the proof is a new "S-procedure" result for averaged integral functionals.

1 Introduction

The stability analysis of general nonlinear systems is one of the central problems in automatic control theory. The pretty general approach consolidating the ideas of system theory (Lyapunov and input-output methods) with the fundamental physics concept dissipation of energy was suggested by J.C.Willems [1]. The notion of dissipative system play the central role in this approach. The term "dissipative" is herein taken to mean that system absorb energy from the environment in some abstract sense. Such "dissipativity" property implies the existence of the so called storage function which can be interpreted as energy stored in system. In turn storage function plays the role of Lyapunov function of the system, and under some further conditions the stability properties of such systems can be demonstrated. In [4] the new class of systems was discribed including the dissipative systems as a special case. The systems of this class were called quasidissipative. Compared to dissipative systems, the quasidissipative systems generally satisfy weaker restrictions on energy transferred to the environment. It was shown in [4] that under some mild additional conditions the trajectories of such systems are bounded in some sense.

In this paper we further extend the concept of quasidissipativity requiring dissipation inequality to be valid only along some sequence of time instants $\{T_j\}, j = 1, 2, \ldots, T_j \to \infty$ when $j \to \infty$. This property is introduced in Section 2 and called *sequential quasidissipativity*. In Section 3 the existence of storage function for sequentially quasidissipative systems is stated. In Section 5 we establish that for differential systems having convergent trajectories sequential quasidissipativity for several supply rates $\omega_i(\cdot), i = 1, \ldots, N$. is equivalent to that with one supply rate $\omega(\cdot)$ which is convex combination of $\omega_i, i = 1, \ldots, N$. The key point of the proof is a new "S-procedure" result for averaged integral functionals which is formulated in Section 4.

2 Quasidissipative systems

We will consider the dynamical systems in state-space form determined on appropriate set $\mathcal{T}$ of time instants. The system is defined as $\sum_m = \{U, \mathcal{U}, Y, \mathcal{Y}, X, \varphi, r\}$ where

U, Y, X - abstract sets called input range, output range and state space respectively,

$\mathcal{U} = \{u: \mathcal{T} \to \mathcal{U}\}$ - input signal space,

$\mathcal{Y} = \{y: \mathcal{T} \to \mathcal{Y}\}$ - output signal space,

$\varphi: \mathcal{T} \times \mathcal{T} \times \mathcal{X} \times \mathcal{U} \to \mathcal{X}$ - state transition function,

$r: \mathcal{T} \times \mathcal{X} \times \mathcal{U} \to \mathcal{Y}$ - readout function.

It is assumed that the state transition function and readout function satysfy the usual axioms [2, 3].

Following Willems [1] and others, we define a function $w: U \times Y \times R \to R$, which is called *supply rate*. We assume that w satisfy reasonable conditions guaranteed the existence of integral $\int_{t_0}^{t} \omega(u(s), y(s))ds$ for any $t_0, t \in \tau$.

Definition 1 [4, 5]. The system with initial state $x(0) = x_0$ is called *weakly quasidissipative* with respect to supply rate w if $\exists \alpha, \beta \geq 0$ s.t. $\forall t \geq 0$, $\forall u \in \mathcal{U}$

$$\int_0^t \omega(u(s), y(s))ds + \alpha t + \beta \geq 0 \qquad (1)$$

whenever $x(0) = x_0$. If the inequality (1) is true with

$\beta = 0$ then system is called *quasidissipative.*

We need some modified version of above definition.

Definition 2. The system with initial state $x(0) = x_0$ is called *sequentially quasidissipative* with respect to supply rate w and if $\exists\alpha \geq 0$ and there exist a sequence of time instants $\{T_j\}, j = 1, 2, \ldots, \lim_{j\to\infty} T_j = +\infty$ s.t. $\forall u \in \mathcal{U}$ and $\forall j$

$$\int_0^{T_j} \omega(u(s), y(s))ds + \alpha T_j \geq 0 \qquad (2)$$

whenever $x(0) = x_0$.

The introduced concept is close to (t_0, T)-dissipativity defined in [14]. We may introduce another definition for sequential quasidissipativity as follows.

Definition 2'. The system with initial state $x(0) = x_0$ is called *sequentially quasidissipative* with respect to supply rate w if $\exists\beta \geq 0$ and there exist a sequence of time instants $\{T_{j'}\}, j = 1, 2, \ldots, \lim_{j'\to\infty} T_{j'} = +\infty$ s.t. $\forall u \in \mathcal{U}$ and $\forall j'$

$$\lim_{j'\to\infty} \frac{1}{T_{j'}} \int_0^{T_j} \omega(u(s), y(s))ds \geq -\beta \qquad (3)$$

(limit in (3) may be finite or infinite). It can be shown that Definitions 2 and 2' are equivalent but for the same system the sequence $\{T_j\}$ may be different from $\{T_{j'}\}$.

Definition 3. Let $\{\omega_i(u, y)\}, i = 1, 2, \ldots, n$ be a set of supply rates. System $\sum_m$ is called *sequentially quasidissipative with respect to supply rates* $\omega_i, i = 1, \ldots, N$, if there exist nonnegative constants $\alpha_i, \quad i = 1, \ldots, N$ and there exist a sequence of time instants $\{T_j\}, j = 1, 2, \ldots$, $\lim_{j\to\infty} T_j = +\infty$, s.t. $\forall u \in \mathcal{U}$ and $\forall j$ the inequality

$$\int_0^{T_j} \omega_i(u(s), y(s))ds + \alpha_i T_j \geq 0 \qquad (4)$$

is valid for some $i \in \{1, 2, \ldots, n\}$.

3 Storage functions for quasidissipative and sequentially quasidissipative systems

We will consider the function $V: X \times \tau \to R$ defined by the expression

$$V(x_0, 0) = -\inf_{\substack{u\in\mathcal{U}, t\geq t_0 \\ x(0)=x_0}} \left(\int_0^t \omega(u(s), y(s))ds + \alpha t \right). \qquad (5)$$

Definition 4. The pair (x_1, t_1) is called *reachable* from $(x_0, 0)$ if $\exists u \in \mathcal{U}$, s.t.

$$\varphi(t_1, 0, x_0, u) = x_1. \qquad (6)$$

In [5] the following results about existence of storage function was obtained.

Theorem 1. Let the system with initial state $x(0) = x_0$ be weakly quasidissipative, and (x_1, t_1) be reachable from $(x_0, 0)$. Then

$$V(x_1, t_1) \leq V(x_0, 0) + \int_0^{t_1} \omega(u(s), y(s))ds + \alpha t_1. \qquad (7)$$

for any $u \in \mathcal{U}$ satisfying the condition (6).

The analog of function (5) for sequentially quasidissipative systems is defined by the expression

$$V(x_0, 0) = -\inf_{\substack{u\in\mathcal{U}, j=0,1,\ldots \\ x(0)=x_0}} \left(\int_0^{T_j} \omega(u(s), y(s))ds + \alpha T_j \right), \qquad (8)$$

where $T_0 = 0$.

The following theorem is the analog of Theorem 1 for sequentially quasidissipative systems.

Theorem 2. Let the system with initial state $x(0) = x_0$ be weakly sequentially quasidissipative, and (x_1, T_k) be reachable from $(x_0, 0)$, where T_k is the element of sequence $\{T_j\}$ from the definition 2. Then

$$V(x_1, T_k) \leq V(x_0, 0) + \int_0^{T_k} \omega(u(s), y(s))ds + \alpha T_k. \qquad (9)$$

for any $u \in \mathcal{U}$ satisfying the condition (6).

Proof. We can write

$$\inf_{u\in U_e, T_j\geq 0} \left(\int_0^{T_j} \omega(u(s), y(s))ds + \alpha T_j \right)$$

$$\leq \inf_{\substack{u\in U_e, 0\leq k\leq j \\ \varphi(T_k,0,x_0,u)=x_1}} \left(\int_0^{T_j} \omega(u(s), y(s))dt + \alpha T_j \right).$$

Taking into account that the trajectories of dynamical system on interval $[T_k, T_j]$ depend only on $x(T_k)$ and $u_{[T_k,T_j]}$ (but not on $u_{[0,T_k]}$) we can write

$$\inf_{\substack{u\in U_e, 0\leq k\leq j \\ \varphi(T_k,0,x_0,u)=x_1}} \left(\int_0^{T_j} \omega(u(s), y(s))dt + \alpha T_j \right)$$

$$= \inf_{\substack{u\in U_e, \\ \varphi(T_k,0,x_0,u)=x_1}} \left(\int_0^{T_k} \omega(u(s), y(s))ds + \alpha T_k \right)$$

$$+ \inf_{\substack{u\in U_e, j\geq k \\ x(T_k)=x_1}} \left(\int_{T_k}^{T_j} \omega(u(s), y(s))ds + \alpha(T_j - T_k) \right)$$

$$\leq \int_0^{T_k} \omega(u(s), y(s))ds + \alpha T_k$$

$$+ \inf_{\substack{u \in U_e, j \geq k \\ x(T_k)=x_1}} \left(\int_{T_k}^{T_j} \omega(u(s), y(s))ds + \alpha(T_j - T_k) \right).$$

Combining in terms of (8) we obtain

$$-V(x_0, 0) \leq \int_0^{T_k} \omega(u(s), y(s))ds + \alpha T_k - V(x_1, T_k).$$

The statement of Theorem 2 follows immediately.

Note, that in the case $\alpha = 0$ the expressions (7) and (9) turn into well-known dissipation inequality for dissipative systems (see for example [1, 2, 3]). Hence, the statements of Theorems 1 and 2 allow to interpret the functions (5) and (8) as storage functions for quasidissipative and sequentially quasidissipative systems respectively.

4 S-procedure for averaged integral functionals

The important role in the modern theory of nonlinear and robust control is played by a trick which was first used in absolute stability theory in 1960s and was called S-procedure by V.A.Yakubovich. Let $\varphi_1(x), \varphi_2(x), \ldots, \varphi_n(x)$ be real functionals on some set X and the following condition is valid:

$$\varphi_1(x) \geq 0 \quad \text{for all} \quad x \in X$$

$$\text{such that} \quad \varphi_2(x) < 0, \ldots, \varphi_n(x) < 0 \tag{10}$$

Definition 6. We say that S-procedure for functionals $\varphi_1(x), \varphi_2(x), \ldots, \varphi_n(x)$ is *loseless* if (10) implies the following condition:

$$\exists \tau_i \geq 0, \quad \sum_{i=1}^{n} \tau_i > 0,$$

$$\forall x \in X \quad \sum_{i=1}^{n} \tau_i \varphi_i(x) \geq 0. \tag{11}$$

It is obvious that (11) implies (10). In case when S-procedure is loseless statements (10) and (11) are equivalent. The loselessness of S-procedure is closely connected with duality relations in some nonlinear extremal problem. It is well known that duality theorem in nonlinear programming is valid for convex extremal problems. However applications in absolute stability, robust control and optimal control involve functionals which are not convex. Also when we use concepts of dissipativity and quasidissipativity for investigation of the nonlinear systems the associated integral functionals are not convex. Therefore we need conditions of loselessness of S-procedure for several nonconvex functionals. Note that S-procedure is loseless for arbitrary two quadratic functionals defined on arbitrary set [6]. However for more than two functionals this statement is not true in general. First result on loselessness of S-procedure for more than two convex functionals was established in [9, 10] for the case when $\varphi_1(x), \varphi_2(x), \varphi_3(x)$ are three quadratic functionals defined on complex linear space. The important results about loselessness of S-procedure for integral quadratic functionals were obtained recently in [12]. Below the new theorem about S-procedure is given appropriate for investigation of quasidissipativity of systems with several supply rates.

Consider dynamical system

$$\dot{x} = f(x, u), \qquad y = h(x, u) \tag{12}$$

where $x \in R^n$, $u \in R^m$, $y \in R^l$ and $f(\cdot)$, $h(\cdot)$ are smooth vector-functions of corresponding dimensions. Let U be a set of piecewise continious bounded functions on $[0, \infty)$ with value in R^m. Suppose system (12) for all $u(\cdot) \in U$ has the following properties.

Property A (convergence property, see [11]): for any $u(\cdot) \in U$ there exist unique bounded on $[0, \infty)$ solution $x_u(t)$ to (12) which is asymptotically stable: any other solution $x(t)$ tends to $x_u(t)$:

$$\lim_{t \to \infty} \|x(t) - x_u(t)\| = 0. \tag{13}$$

Suppose also that convergence in (13) is uniform over any bounded set of initial conditions of (12).

Let $g_i(x, u), j = 1, \ldots, l$ be real functions on $R^n \times R^m$ which are uniformly continious on any bounded set.

Property B: $\forall u \in U$ the limits

$$\lim_{j \to \infty} \frac{1}{T_j} \int_0^{T_j} g_i(x(s), u(s))ds, \qquad i = 1, \ldots, n,$$

where $x(t)$ is some solution of system (12), $\lim T_j = +\infty$, exist and don't depend on sequence $\{T_j\}$.

Then we can define functionals φ_i on U, $i = 1, \ldots, n$ as follows:

$$\varphi_i(u(\cdot)) = \lim_{j \to \infty} \frac{1}{T_j} \int_0^{T_j} g_i(x(s), u(s))ds, \qquad i = 1, \ldots, n. \tag{14}$$

Due to convergence property the values of functionals (14) don't depend on initial condition $x(0)$.

The following theorem is valid.

Theorem 3. Let the system (12) has the properties A and B. Then S-procedure for any number of functionals (14) is loseless.

The key point of the proof of this theorem is the following auxiliary statement, which is interesting as itself.

Lemma. Introduce for any $u \in U$ the following vector $\Phi(u) = [\varphi_1(u), \ldots, \varphi_N(u)] \in R^N$ and denote $\Phi(U) = \{\Phi(u), u \in U\} \subset R^N$. Then the closure of set $\Phi(U)$ is the convex set.

Proof of the Lemma. It is suffice to prove that $\forall u_1 \in U,\ u_2 \in U$

$$\frac{z_1 + z_2}{2} \in \overline{\Phi},$$

where $\overline{\Phi}$ is the closure of Φ, $z_i = \mathrm{col}(\phi_1(u_i(\cdot)), \ldots, \phi_l(u_i(\cdot)))$, $i = 1, 2$, that is there exists a sequence $w_n \in \Phi : w_n \to \frac{z_1+z_2}{2}$ when $n \to \infty$. Fix $T > 0$ and define $v_n(\cdot)$ by the expression

$$v_n(t) = \begin{cases} u_1(t - kT) & \text{if } 0 \le t < nT, \\ u_2(t - kT) & \text{if } nT \le t < 2nT \end{cases}$$

for $0 \le t < 2nT$, $v_n(t) = v_n(t - 2nT)$ for $t \ge 2nT$, and denote $w_n = [\varphi_1(v_n), \ldots, \varphi_N(v_n)]$.

It is necessary to prove that for any $\epsilon > 0$ there exists N_ϵ s.t.

$$\|w_n - \frac{z_1 + z_2}{2}\| < \epsilon \quad \text{when} \quad n > N_\epsilon.$$

Because of the continuity of the function $g_j(x, u)$ $\forall \epsilon_1 > 0\ \exists \delta > 0$, s.t.

$$\|g_j(x, u) - g_j(y, u)\| < \epsilon_1, \quad j = 1, \ldots, l,$$

when $\|x - y\| < \delta$. From the convergence property it follows that $\exists \tau_\delta > 0$, s.t.

$$\|x_{u_i}(t) - x^0_{u_i}(t)\| < \delta \quad \text{when} \quad t > \tau_\delta \quad i = 1, 2.$$

It may considered that $\tau_\delta = k_\delta T$, where k_δ is integer number. For the purpose of estimation of $\phi_j(v_n(\cdot))$ for $n > k_\delta$ we devide the set of integer numbers into three parts P, Q_1, Q_2 where

$$P = \{t : kT \le t \le (k+1)T,\ k \in \{\{0, 1, \ldots, k_\delta\}$$
$$\cup \{n, \ldots, n + k_\delta\} \cup \{2n, \ldots, 2n + k_\delta\} \cup \ldots\}\},$$
$$Q_1 = \{t : kT \le t \le (k+1)T,\ k \in \{\{k_\delta + 1, \ldots, n\}$$
$$\cup \{2n + k_\delta + 1, \ldots, 3n\} \cup \ldots\}\},$$
$$Q_2 = \{t : kT \le t \le (k+1)T,$$
$$k \in \{\{n + k_\delta + 1, \ldots, 2n\}$$
$$\cup \{3n + k_\delta + 1, \ldots, 4n\} \cup \ldots\}\}.$$

Obviously P is a totality of transient intervals. Clearly $v_n(t) = u_i(t)$ when $t \in Q_i$, $i = 1, 2$. Let $t = 2dn$ where d is natural number and denote $P^t = P \cap [0, t]$, $Q_i^t = Q_i \cap [0, t]$. Then

$$\frac{1}{t}\int_0^t g_j(x_{u_i}(s), v_n(s))ds$$

$$= \frac{1}{t}\left[\int_{P^t} g_j(\cdot)ds + \int_{Q_1^t} g_i(\cdot)ds + \int_{Q_2^t} g_j(\cdot)ds\right]$$

$$= I_0 + I_1 + I_2$$

It is felt that the bounded set of initial conditions Ω include the ϵ-neighbourhood of the solutions $x^0_{u_i}(s)$, $0 \le s < +\infty$, $i = 1, 2$. Moreover due to the convergence property and the fact that the solution of system depend continiously on initial conditions $\exists \mathcal{D} > 0$ s.t. $\forall s \le t$

$$\|x_{u_i}(s)\| \le \mathcal{D}$$

and because of the continuity of g_i there exist $\mathcal{D}_g$ s.t. $\forall s \le t$

$$\|g_j(x_{u_i}(s), u_i(s))\| \le \mathcal{D}_g \quad i = 1, 2.$$

Then in the end of any interval $[knT, (k+1)nT]$ we have $x_{v_n}(t) \in \Omega$. Therefore

$$|I_0| \le \frac{\mathcal{D}_g(k_\delta + 1)}{n}$$

and

$$|I_0| \le \frac{\epsilon}{5} \quad \text{when} \quad n \ge N_\epsilon = \frac{5\mathcal{D}_g(k_\delta + 1)}{\epsilon}.$$

Furthermore

$$I_i = \frac{1}{t}\int_{Q_i^t} g_j(x_{u_i}(s), u_i(s))ds$$

$$= \frac{1}{t}\int_{Q_i^t} g_j(x^0_{u_i}(s), u_i(s))ds$$

$$+\frac{1}{t}\int_{Q_i^t} [g_j(x_{u_i}(s), u_i(s)) - g_j(x^0_{u_i}(s), u_i(s))]ds$$

$$= \frac{\varphi_j(u_i(\cdot))}{2} + M_{ij}$$

where

$$|M_{ij}| \le \frac{\mathcal{D}_g(k_\delta + 1)}{n} + \frac{\epsilon_1(1 - \frac{k_\delta+1}{n})}{2}$$
$$\le \frac{\epsilon}{5} + \frac{\epsilon_1}{2}.$$

By choosing ϵ_1 s.t. $\frac{\epsilon_1 T}{2} < \frac{\epsilon}{5}$ we obtain that

$$\left|\varphi_j(v_n) - \frac{\varphi_j(u_1(\cdot)) + \varphi_j(u_2(\cdot))}{2}\right| < \epsilon$$

when $n > N_\epsilon$.

$$\|w_n - \frac{z_1 + z_2}{2}\| \le \sqrt{N} \cdot \epsilon.$$

The statement of Theorem 3 is derivable from Lemma in the regular way (see [9]).

5 Conditions of sequential quasidissipativity for several supply rates

Now we are in position to formulate the main result of the paper.

Theorem 4. System (12) is weakly sequentially quasidissipative with respect to supply rates $\{\omega_i(u(t),y(t))\}, i=1,2,\ldots,n$ if and only if there exist constants $\tau_i \geq 0, i=1,2,\ldots,n$, $\sum_{i=1}^n \tau_i > 0$, s.t the system is weakly sequentially quasidissipative with respect to supply rate defined by

$$\omega_\tau(u(t),y(t)) = \sum_{i=1}^n \tau_i \omega_i(u(t),y(t)).$$

Proof. We have that there exist nonnegative constants $\alpha_i, \quad i=1,\ldots,N$ and $\forall u \in \mathcal{U}$ there exist a sequence of time instants $\{T_j\}, j=1,2,\ldots, \lim_{j\to\infty} T_j = +\infty$, s.t. $\forall j$ the inequality

$$\int\limits_0^{T_j} \omega_i(u(s),y(s))ds + \alpha_i T_j \geq 0 \qquad (4)$$

for some $i \in \{1,2,\ldots,n\}$. Because of the n is finite number then for some $i' \in \{1,2,\ldots,n\}$ the subsequence $\{T_k\}$ are derivable s.t. $\forall k$ we have

$$\int\limits_0^{T_k} \omega_{i'}(u(s),y(s))ds + \alpha_{i'} T_k \geq 0. \qquad (4)$$

In view of equivalence of definitions 2 and 2' it immediately follows that there exist a sequence of time instants $\{T_{k'}\}, k'=1,2,\ldots, \lim_{k'\to\infty} T_{k'} = +\infty$, s.t.

$$\lim_{k'\to\infty} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \omega_{i'}(u(s),y(s))ds \geq -\beta_{i'}$$

for some $\beta_{i'} \geq 0$. It follows that

$$\lim_{k'\to\infty} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} (\omega_i(u(s),y(s)) + \beta_i + \epsilon)ds > 0$$

$\forall \epsilon > 0$. Throudh the use of S-procedure it can be obtained that

$$\lim_{k'\to\infty} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \sum_{i=1}^n \tau_i(\omega_i(u(s),y(s)) + \beta_i + \epsilon)ds > 0$$

for some $\beta_i \geq 0 \quad ,i=1,2,\ldots,n$. In view of arbitrarity of $\epsilon > 0$ and nonnegativity of all τ_i it follows that

$$\lim_{k'\to\infty} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \sum_{i=1}^n \tau_i \omega_i(u(s),y(s))ds \geq -\sum_{i=1}^n \tau_i \beta_i. \qquad (15)$$

The converse statement can be proved as follows. Consider the sequences

$$\{\xi_{k'}\}_i = \frac{\tau_i}{T_{k'}} \int\limits_0^{T_{k'}} \omega_i(u(s),y(s))ds,$$

$$i=1,\ldots,n,\ k'=1,2,\ldots. \qquad (16)$$

If there are the unbounded sequences among them then there is at least one number $l \in \{1,\ldots,n\}$ s.t. $\{\xi_{k'}\}_l$ is unbounded from above, otherwise the inequality (15) would be false. Thus it is possible to separate the subsequence of $\{\xi_{k'}\}_{i'}$ which is tend to $+\infty$. Let all sequences (16) are bounded. From the fact that $\forall \tau_i \geq 0$ and in view of the proreties of upper limit it follows that

$$\sum_{i=1}^n \tau_i \overline{\lim_{k'\to\infty}} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \omega_i(u(s),y(s))ds$$

$$\geq \lim_{k'\to\infty} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \sum_{i=1}^n \tau_i \omega_i(u(s),y(s))ds$$

$$\geq -\sum_{i=1}^n \tau_i \beta_i. \qquad (17)$$

If $\tau_i = 0$ for some i then the correspondence terms can be eliminated from inequalities (17). After reindexing rest terms we have

$$\sum_{i=1}^{n'} \tau_i \overline{\lim_{k'\to\infty}} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \omega_i(u(s),y(s))ds \geq -\sum_{i=1}^{n'} \tau_i \beta_i$$

where all $\tau_i > 0$, $n' \leq n$. Since the right part of the last inequality is nonpositive then $\exists m \in \{1,2,\ldots,n'\}$ s.t.

$$\tau_m \overline{\lim_{k'\to\infty}} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \omega_m(u(s),y(s))ds \geq -\sum_{i=1}^{n'} \tau_i \beta_i$$

and

$$\overline{\lim_{k'\to\infty}} \frac{1}{T_{k'}} \int\limits_0^{T_{k'}} \omega_m(u(s),y(s))ds \geq -\frac{\sum_{i=1}^n \tau_i \beta_i}{\tau_m},$$

From definition of upper limit it follows that there exist the subsequence $\{T_{k"}\} \subset \{T_{k'}\}$ s.t.

$$\lim_{k"\to\infty} \frac{1}{T_{k"}} \int\limits_0^{T_{k"}} \omega_m(u(s),y(s))ds \geq -\frac{\sum_{i=1}^n \tau_i \beta_i}{\tau_m},$$

The proof is complete.

6 Conclusions

The meaning of the above results is expanding the scope of applications for dissipativity-like concepts. Now these concepts can be used for studying not only Lyapunov and asymptotic stability as in [2, 3, 13] but also for examination of boundedness of system trajectories.

On the other hand Theorem 4 demonstrates that further complication of quasidissipativity by means several supply rates may appear to be useless (see also [15]). It may happen if the corresponding version of S-procedure is loseless. Finally the S-procedure losslesness theorem allows to solve new class of optimisation problems using approach of [7].

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