Adaptive Nonlinear Partial Observers with Application to Time-Varying Chaotic Systems

Efimov D. V., Fradkov A. L., *Senior Member, IEEE*

***Abstract* — New applicability conditions for adaptive nonlinear observer are developed, which provide partial observation of uncertain nonlinear affine plant with estimation of unknown parameters. Applicability of the proposed results to time-varying chaotic systems is demonstrated by examples of brusselator and Duffing systems. Computer simulation results are presented.**

***Index Terms*— adaptive observer, partial stability, synchronization, information transmission.**

I. Introduction

Adaptive observers design for nonlinear systems was extensively studied during last decade. Such an interest was mainly motivated by possible application of adaptive observers to information encoding and transmission. Typically a chaotic dynamical system is used as a transmitter and its output signal is changed by modulating its parameters [11, 12, 13]. It was shown in [7] that it is possible to build a receiver based on adaptive observer, which can track output of transmitter and estimate transmitter parameters under some mild conditions. Potentialities of fast information transmission in the presence of noise in such systems were demonstrated in [1, 2, 3]. Several techniques were previously used to design receivers [4, 5, 7, 14, 15, 17, 21], most of them being based on passifiability property of transmitter under assumption that relative degree of transmitter is equal to zero or one. Other solutions can be found in [9, 10], where a state feedback was used for adaptive observer construction and robust properties of proposed schemes were not investigated. Recent paper [8] overcame the relative degree limitation for adaptive observer-based communication systems and extended them to a class of nonpassifiable systems. The result of [8] is based on a new canonical form of nonlinear adaptive observers [6, 18].

In the present paper the result of [8] is extended to the case of partial observation when exact estimation of only a part of the transmitter state variables is needed. For such a case the applicability conditions are obtained, which allow to enlarge class of transmitter systems. Unlike previous results, our results allow to use time-varying systems for both transmitter and receiver. For example, the proposed results are applicable to chaos generators with external periodic excitation. In the paper the obtained results are demonstrated by two examples of practical importance: the brussellator model [19] and the Duffing system excited by harmonic signals.

In Section 2 the brussellator model [19] and the Duffing system are introduced. In Section 3 an adaptive observer scheme is designed under assumptions covering case of the systems from the previous section. Computer simulation results are presented in Section 4.

Both authors are with the Institute of Problem of Mechanical Engineering, Control of Complex Systems laboratory, Bolshoi av., 61, V.O. , St-Petersburg, 199178 Russia (e-mails: efde@mail.rcom.ru, alf@ccs1.ipme.ru).

This work is partly supported by grants 02-01-00765 and 03-01-06373 of Russian Foundation for Basic Research.

II. Statement of the Problem

In the literature on chaos and its applications two main classes of chaotic systems are usually considered: autonomous (time-invariant) and non-autonomous (time-varying) (see, e.g.[16]). However, in applications to information transmission in most cases only the former ones are used [11,12,13]. At the same time time-varying systems are sometimes easier to implement, or, they can be modeled by using other physical principles. Two typical examples of nonlinear time-varying systems where chaos is generated by means of applying a harmonic excitation signal are brusselator, modeling some chemical reactions and Duffing system, used in many studies in mechanics.

We will use the equations of forced brusselator model in the following form [19]:

$$\begin{aligned} \dot{s}_1 &= A + a\cos(\omega t) - (B+1+\theta)s_1 + s_1^2 s_2; \\ \dot{s}_2 &= (B+\theta)s_1 - s_1^2 s_2;\ y = s_1, \end{aligned} \tag{1}$$

where s_1 and s_2 are state variables with positive real values; y as usually is on-line measured output; parameters $A=0.4$, $a=0.05$, $\omega=0.81$ and $B=1.2$; unknown or "transmitted" parameter θ belongs to set $\Omega_\theta=[0,2]$ (during simulation it will be taken $\theta=1$). We will also use the Duffing's system equations in the form [16]:

$$\begin{aligned} \dot{x}_1 &= x_2;\ y = x_1; \\ \dot{x}_2 &= -a x_1 + \theta x_1^3 + B\cos(\omega t), \end{aligned} \tag{2}$$

where x_1 and x_2 are state variables; y is measured output; $\theta\in\Omega_\theta=[0.5,1.5]$ is "transmitted" parameter; model parameters $a=B=\omega=1$.

The problem is to design a dynamical system (adaptive observer) some variables of which provide estimates of a specified part of variables and parameters of the transmitter (system (1) or (2)). Since the systems are time-varying, it is not always possible to achieve zero estimation error. Therefore, we will be interested in achieving bounded and sufficiently small asymptotic value of the estimation error.

III. Design of adaptive observers

Following [8] we assume that model of transmitter system can be written as follows:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{y})\mathbf{x} + \varphi(\mathbf{y}) + \mathbf{B}(\mathbf{y})\boldsymbol{\theta},\ \mathbf{y} = \mathbf{C}\mathbf{x}\,, \tag{3}$$

where $\mathbf{x} \in R^n$ is state space vector of transmitter; $\mathbf{y} \in R^m$ is output vector; $\boldsymbol{\theta} \in \Omega_\theta \subset R^p$ is vector of "unknown" parameters of transmitter, or, better to say, it is transmitted vector, which values belonged to some known compact set Ω_θ should be estimated by receiver basing on current measurements of transmitter output $\mathbf{y}$. Vector function φ and columns of matrix functions $\mathbf{A}$ and $\mathbf{B}$ are locally Lipschitz continuous, $\mathbf{C}$ is some constant matrix of appropriate dimension. Thus, for any initial condition $\mathbf{x}_0 \in \Omega_x \subset R^n$ and any $\boldsymbol{\theta} \in \Omega_\theta$ (where Ω_x some known, probably compact, set), solution of (3) $\mathbf{x}(t, \mathbf{x}_0, \boldsymbol{\theta})$ is well defined at the least locally (further we will omit dependence of $\mathbf{x}_0$ and $\boldsymbol{\theta}$ if it is clear from the context and will simply write $\mathbf{x}(t)$). For transmitter it is naturally to suppose [8], that its solution is bounded and defined for all $t \geq 0$.

A s s u m p t i o n 1. *For any initial conditions $\mathbf{x}_0 \in \Omega_x$ and any $\boldsymbol{\theta} \in \Omega_\theta$, solution of (3) $\mathbf{x}(t, \mathbf{x}_0, \boldsymbol{\theta})$ is an essentially bounded function of time:*

$$|\mathbf{x}(t, \mathbf{x}_0, \boldsymbol{\theta})| \leq \sigma_0(|\mathbf{x}_0|),\ \sigma_0 \in \mathcal{K} \textit{ for all } t \geq 0\,. \qquad \blacksquare$$

As usually, it is said, that function $\rho : R_{\geq 0} \to R_{\geq 0}$ belongs to class $\mathcal{K}$, if it is strictly increasing and $\rho(0) = 0$; $\rho \in \mathcal{K}_\infty$ if $\rho \in \mathcal{K}$ and $\rho(s) \to \infty$ for $s \to \infty$ (radially unbounded). Function $\mathbf{x} : R_{\geq 0} \to R^n$ is essentially bounded, if

$$\|\mathbf{x}\| = ess\sup\{|\mathbf{x}(t)|, t \geq 0\} < +\infty\,,$$

where $|\cdot|$ denotes usual Euclidean norm. Such assumption is valid for class of system (3) with so-called chaotic dynamics. To design an observer we need also two assumptions, which deal with stabilizability property of linear part of transmitter system (3).

A s s u m p t i o n 2. *There exists continuous matrix function $\mathbf{K} : R^m \to R^{n\times m}$, such, that there exists function $V : R^n \to R_{\geq 0}$,*

$$\alpha_1(|\mathbf{C}\mathbf{x}|) \leq V(\mathbf{x}) \leq \alpha_2(|\mathbf{C}\mathbf{x}|),\ |\partial V/\partial \mathbf{x}| \leq \rho(|\mathbf{x}|);$$

$$\partial V(\mathbf{x})/\partial \mathbf{x}\, \mathbf{G}(\mathbf{y})\mathbf{x} \leq -\alpha_3(|\mathbf{C}\mathbf{x}|) + \alpha_4(|\mathbf{x}|),$$

for any bounded values of $\mathbf{y} \in R^m$ and $\mathbf{x} \in R^n$, where α_1, α_2, α_3 are some functions from class $\mathcal{K}_\infty$ and α_4, ρ are functions from class $\mathcal{K}$, matrix $\mathbf{G}(\mathbf{y}) = \mathbf{A}(\mathbf{y}) - \mathbf{K}(\mathbf{y})\mathbf{C}$. $\blacksquare$

A s s u m p t i o n 3. *For any initial conditions $\mathbf{s}_0 \in R^n$ solution of system*

$$\dot{\mathbf{s}} = \mathbf{G}(\mathbf{y})\mathbf{s} + \mathbf{r}\,, \tag{4}$$

is bounded for any essentially bounded inputs $\mathbf{r}$ and $\mathbf{y}$:

$$|\mathbf{s}(t, \mathbf{s}_0, \mathbf{r})| \leq \sigma_1(|\mathbf{s}_0|) + \sigma_1(\|\mathbf{r}\|),\ \sigma_1 \in \mathcal{K} \textit{ for all } t \geq 0\,. \qquad \blacksquare$$

By itself Assumption 2 means nothing, but with combination with Assumption 3 they provide for system (4) ultimate boundedness of signal $\mathbf{C}\mathbf{s}$. Indeed, Assumption 3 implies existence of finite norm $\|\mathbf{s}\|$ for state vector of system (4), then inequality for time derivative of function V from Assumption 2 takes form:

$$\dot{V} \leq -\alpha_3 \circ \alpha_2^{-1}(V(\mathbf{s})) + \alpha_4(\sigma_1(|\mathbf{s}_0|) + \sigma_1(\|\mathbf{r}\|))\,.$$

From the last inequality the following output asymptotic gain can be obtained:

$$\lim_{t\to+\infty} |\mathbf{C}\mathbf{s}(t)| \leq \lambda_1(|\mathbf{s}_0|) + \lambda_2(\|\mathbf{r}\|),$$

$$\lambda_1(s) = \alpha_4(2\sigma_1(s)) + 0.5\rho(2\sigma_1(s))^2\,,$$

$$\lambda_2(s) = 0.5s^2 + \alpha_4(2\sigma_1(s)) + \rho(2\sigma_1(s))^2 s\,.$$

If the gain function α_4 can be "decreased" by appropriate choice of design matrix function $\mathbf{K}$, then the desired asymptotic bound for signal $\mathbf{C}\mathbf{s}$ can be assigned.

It is worth to stress, that comparing with [8] Assumptions 2 and 3 enlarge the class of models for transmitter systems, since they do not suppose global asymptotic stability property of system (4) uniformly with respect to input $\mathbf{y}$ and $\mathbf{r} = 0$, as it was done in [8]. Before we prove our main result we should introduce a new property.

D e f i n i t i o n 1. *Function $a : R_{\geq 0} \to R$ is called (μ, Δ)–positive in average (PA), if for any $t \geq 0$ and any $\delta \geq \Delta$, $\mu > 0$,*

$$\int_t^{t+\delta} a(\tau)d\tau \geq \mu\delta\,. \qquad \blacksquare$$

In other words, time function $a(t)$ is (μ, Δ)–PA, if its average value a_{av} on any large enough time interval $[t, t+\delta]$, $\delta \geq \Delta$,

$$a_{av} = \frac{1}{\delta}\int_t^{t+\delta} a(\tau)d\tau$$

is not smaller than some positive constant μ. Note, that function $a(t) = \sin(t) + \alpha$ admits this property for any strictly positive constant α, thus, function a should not be positive for all $t \geq 0$. We will need the following lemma that can be easily proved by integration.

L e m m a 1. *Let us consider time-varying linear dynamic system*

$$\dot{p} = -a(t)p + b(t),\ t_0 \geq 0\,, \tag{5}$$

where $p \in R$, $p(t_0) \in R$ and functions $a : R_{\geq 0} \to R$, $b : R_{\geq 0} \to R$ are Lebesgue measurable and essentially bounded. Then:

A. *Solution of system (5) is defined for all $t \geq t_0$:*

$$|p(t)| \leq \left[|p(t_0)| + \|b\|e^{\|a\|t}\right]e^{\|a\|(t-t_0)}\,.$$

B. *If function a is (μ, Δ)–PA for some $\mu > 0$, $\Delta > 0$, then this solution is bounded and the following upper estimate* *holds*:

$$p(t)\le\begin{cases}\left[|p(t_0)|+\|b\|e^{\|a\|(t_0+\Delta)}\right]e^{\|a\|\Delta} & \text{if } t\le t_0+\Delta;\\ p(t_0)e^{-\mu(t-t_0)}+\mu^{-1}\|b\|e^{-\mu(\Delta+t_0)}+ & \\ +\|a\|^{-1}\|b\|e^{-\|a\|(t-\Delta)} & \text{if } t\ge t_0+\Delta,\end{cases}$$

additionally, if $b(t)\to 0$ *for* $t\to+\infty$, *then also* $p(t)\to 0$ *asymptotically*. ■

In this work we will use the same adaptive observer equations as in paper [8]:

$$\dot{\mathbf{z}}=\mathbf{A}(\mathbf{y})\mathbf{z}+\varphi(\mathbf{y})+\mathbf{B}(\mathbf{y})\boldsymbol{\theta}+\mathbf{K}(\mathbf{y})(\mathbf{y}-\hat{\mathbf{y}}),\ \hat{\mathbf{y}}=\mathbf{C}\mathbf{z}; \tag{6}$$

$$\dot{\boldsymbol{\eta}}=\mathbf{G}(\mathbf{y})\boldsymbol{\eta}-\boldsymbol{\Omega}\dot{\hat{\boldsymbol{\theta}}}; \tag{7}$$

$$\dot{\boldsymbol{\Omega}}=\mathbf{G}(\mathbf{y})\boldsymbol{\Omega}+\mathbf{B}(\mathbf{y}); \tag{8}$$

$$\dot{\hat{\boldsymbol{\theta}}}=\gamma\boldsymbol{\Omega}^T\mathbf{C}^T(\mathbf{y}-\hat{\mathbf{y}}+\mathbf{C}\boldsymbol{\eta}), \tag{9}$$

where $\mathbf{z}\in R^n$ is vector of estimates of nonmeasurable state space vector of system (3); $\hat{\mathbf{y}}\in R^m$ is vector of on-line measurable output $\mathbf{y}$ estimates; vector $\boldsymbol{\eta}\in R^n$ and matrix $\boldsymbol{\Omega}\in R^{n\times p}$ are auxiliary variables, which help to overcome high relative degree obstruction; $\hat{\boldsymbol{\theta}}\in R^p$ is vector of estimates of "transmitted" vector $\boldsymbol{\theta}$; $\gamma>0$ is a design parameter.

Theorem 1. *Let Assumptions 1, 2 and 3 hold and function* $|\mathbf{C}\,\boldsymbol{\Omega}(t)|^2$ *satisfies* (μ,Δ)*–PA condition for some* $\mu>0$, $\Delta>0$, *then closed-loop system consisting of transmitter (3), adjustable receiver (6), scheme of augmentation (7), (8) and adaptation algorithm (9) provides boundedness property of the system solution for any initial conditions and any* $\gamma>0$, *additionally*

$$\lim_{t\to+\infty}\left|\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}(t)\right|\le\sqrt{\mu^{-1}e^{-\gamma\mu\Delta}}\,\lambda(|\boldsymbol{\delta}_0|).$$

Proof. We will denote $\mathbf{e}=\mathbf{x}-\mathbf{z}$ as state estimation error of proposed observer (6). Let $\boldsymbol{\varepsilon}=\mathbf{y}-\hat{\mathbf{y}}$ be corresponding measurable output error. The behavior of error $\mathbf{e}$ can be rewritten as follows:

$$\dot{\mathbf{e}}=\mathbf{G}(\mathbf{y})\mathbf{e}+\mathbf{B}(\mathbf{y})(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}),\ \boldsymbol{\varepsilon}=\mathbf{C}\mathbf{e}. \tag{10}$$

Introduce the auxiliary error signal

$$\boldsymbol{\delta}=\mathbf{e}+\boldsymbol{\eta}-\boldsymbol{\Omega}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}),$$

which dynamics coincides with auxiliary system (4) with zero input $\mathbf{r}$:

$$\dot{\boldsymbol{\delta}}=\mathbf{G}(\mathbf{y})\boldsymbol{\delta}. \tag{11}$$

Hence, according to previous discussion, signal $\boldsymbol{\delta}$ is bounded and signal $\mathbf{C}\boldsymbol{\delta}$ is ultimately bounded. Let us analyze the following Lyapunov function candidate with respect to part of variables [20]:

$$W(\hat{\boldsymbol{\theta}})=\gamma^{-1}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^T(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}),$$

its time derivative for system (9) takes form:

$$\dot{W}=-2(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^T\boldsymbol{\Omega}^T\mathbf{C}^T\mathbf{C}(\boldsymbol{\delta}+\boldsymbol{\Omega}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}))\le$$
$$\le-|\mathbf{C}\,\boldsymbol{\Omega}|^2|\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}|^2+|\mathbf{C}\boldsymbol{\delta}|^2=-\gamma|\mathbf{C}\,\boldsymbol{\Omega}|^2W+|\mathbf{C}\boldsymbol{\delta}|^2.$$

According to Lemma 1 solution of the last linear time-varying inequality is bounded, if functions $\gamma|\mathbf{C}\,\boldsymbol{\Omega}(t)|^2$ and $|\mathbf{C}\boldsymbol{\delta}(t)|^2$ are. But these two conditions are satisfied, due to form of systems (8), (11) (they are in class of system (4)) and Assumptions 1 and 3, signals $\boldsymbol{\Omega}$ and $\boldsymbol{\delta}$ are bounded and:

$$\gamma|\mathbf{C}\,\boldsymbol{\Omega}(t)|^2\le\gamma|\mathbf{C}|^2(\sigma_1(|\boldsymbol{\Omega}_0|)+\sigma_1(B_{\max}(\mathbf{x}_0)))^2<+\infty,$$

$$|\mathbf{C}\,\boldsymbol{\delta}(t)|^2\le|\mathbf{C}|^2\sigma_1(|\boldsymbol{\delta}_0|)^2<+\infty,$$

where $B_{\max}(\mathbf{x}_0)=\sup\{|\mathbf{B}(\mathbf{C}\mathbf{x})|,\ |\mathbf{x}|\le\sigma_0(|\mathbf{x}_0|)\}$ and expression $|\mathbf{C}|$ for some matrix $\mathbf{C}=\{c_{i,j}\}$, $i=\overline{1,m}$, $j=\overline{1,n}$ ($c_{i,j}$ are corresponding elements of matrix $\mathbf{C}$) should be understood in the following sense:

$$|\mathbf{C}|=\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}c_{i,j}^2}.$$

From Lemma 1, the following estimate asymptotically holds with $t_0=0$:

$$\lim_{t\to+\infty}W(t)\le\gamma^{-1}\mu^{-1}\lambda(|\boldsymbol{\delta}_0|)^2e^{-\gamma\mu\Delta},$$

and parameter error boundedness $\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}(t)$ is also obtained. Note, that system (10) has form of system (4) too with input $\mathbf{B}(\mathbf{y})(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})$. Therefore, the boundedness of signal $\mathbf{e}$ follows from Assumption 3. Finally, signal $\boldsymbol{\eta}$ is bounded because it forms bounded signal $\boldsymbol{\delta}$, and all other signals, which also form signal $\boldsymbol{\delta}$, are bounded. Therefore, boundedness of the system solution is established. ■

Note, that the result of the last theorem does not guarantee attractivity property of output estimation error $\boldsymbol{\varepsilon}$ for an adaptive observer. In general only asymptotic convergence of $\boldsymbol{\varepsilon}$ and parameter error $\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}(t)$ to some compact set is provided. In fact, for information transmitting purposes only parameter error convergence is necessary, and output error estimation is not needed. Signal $|\mathbf{C}\,\boldsymbol{\Omega}(t)|^2$ is directly produced by observer system and its PA parameters (constants μ and Δ) can be directly computed based on its on-line measurements, i.e. it is possible for given positive constant Δ to calculate an average value

$$\Omega_{av}(t)=\frac{1}{t}\int_0^t|\mathbf{C}\,\boldsymbol{\Omega}(\tau)|^2d\tau$$

for $t\ge\Delta$. Then constant μ can be computed as follows:

$$\mu=\chi\inf_{t\ge\Delta}\{\Omega_{av}(t)\},$$

where $\chi\in(0,1]$ is some design constant.

It is worth noticing that asymptotic estimate for parameter error $\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}(t)$, presented in the Theorem, can be evaluated with some difficulties, since the initial condition $\boldsymbol{\delta}_0$ itself depends on unmeasured discrepancy $\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}(0)$, but this estimate helps to understand mechanisms for parameter error decreasing (for example, increasing coefficient γ or decreasing gain function λ).

IV. Applications

In this section we will consider two examples of adaptive observer construction for models of physical systems presented in Section 2, which are useful from practical point of view. Both examples satisfy conditions of Theorem 1, computer simulations demonstrate workability of proposed results.

A. Forced brusselator model

Model (1) has a difference with respect to the studied before system (3), right hand side of this system explicitly depends on time. So, to compensate this shortage and to simplify below discussion let us introduce the following change of coordinates

$$x_1 = s_1\,,\ \ x_2 = s_1 + s_2\,,$$

then equations of the model take form:

$$\begin{aligned} \dot{x}_1 &= A + a\,x_3 - (B+1+\theta)x_1 - x_1^3 + x_1^2\,x_2; \\ \dot{x}_2 &= A + ax_3 - x_1; \\ \dot{x}_3 &= \omega x_4; \\ \dot{x}_4 &= -\omega x_3;\ \ y = x_1, \end{aligned} \tag{12}$$

where new state variables x_3 and x_4 were added to exclude time dependence from system equations and to generate needed sinusoidal signal. It is clear, that for corresponding initial conditions both models produce the same output signal. So, system (12) satisfies conditions of Assumption 1 for any bounded nonnegative initial conditions.

As an observer for system (12) let us analyze the following dynamical system:

$$\begin{aligned} \dot{z}_1 &= A + a\,z_3 - \left(B+\hat{\theta}\right)y - z_1 - y^3 + y^2\,z_2 + K\,(\,y - z_1); \\ \dot{z}_2 &= A + az_3 - z_1; \\ \dot{z}_3 &= \omega z_4; \\ \dot{z}_4 &= -\omega z_3;\ \ \hat{y} = z_1, \end{aligned} \tag{13}$$

where $\hat{\theta}$ is an estimate of θ. For this observer dynamic of state error $\mathbf{e} = \mathbf{x} - \mathbf{z}$ can be rewritten as follows:

$$\begin{aligned} \dot{e}_1 &= a\,e_3 - \left(\theta - \hat{\theta}\right)y - e_1 + y^2\,e_2 - K\,e_1; \\ \dot{e}_2 &= ae_3 - e_1; \\ \dot{e}_3 &= \omega e_4; \\ \dot{e}_4 &= -\omega e_3;\ \varepsilon = e_1\,, \end{aligned}$$

and

$$\mathbf{G}(\,y) = \begin{bmatrix} -K-1 & y^2 & a & 0 \\ -1 & 0 & a & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & -\omega & 0 \end{bmatrix}$$

which admits conditions of Assumption 3 for any non zero values of signal y and $K > 0$. Assumption 2 is satisfied with $V(\,e_1) = 0.5e_1^2$, since its time derivative has an upper bound:

$$\dot{V} \le -2K\,V + 0.5(\,a\,e_3)^3 + 0.5(\,y^2\,e_2^2\,)^2\,.$$

Augmented error systems and adaptation algorithm have form:

$$\begin{aligned} \dot{\Omega}_1 &= a\,\Omega_3 - y - \Omega_1 + y^2\,\Omega_2 - K\,\Omega_1; \\ \dot{\Omega}_2 &= a\Omega_3 - \Omega_1; \\ \dot{\Omega}_3 &= \omega\Omega_4; \\ \dot{\Omega}_4 &= -\omega\Omega_3; \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{\eta}_1 &= a\,\eta_3 - \eta_1 + y^2\,\eta_2 - K\,\eta_1 - \Omega_1\dot{\hat{\theta}}; \\ \dot{\eta}_2 &= a\eta_3 - \eta_1 - \Omega_2\dot{\hat{\theta}}; \\ \dot{\eta}_3 &= \omega\eta_4 - \Omega_3\dot{\hat{\theta}}; \\ \dot{\eta}_4 &= -\omega\eta_3 - \Omega_4\dot{\hat{\theta}}; \end{aligned} \tag{15}$$

$$\dot{\hat{\theta}} = \gamma\Omega_1\,(\,y - z_1 + \eta_1\,)\,. \tag{16}$$

The result of computer simulation of system (12)–(16) is presented in Fig. 1. During simulation all initial conditions were placed as zero except $x_1(0)=1$, $x_2(0)=2$, $x_3(0)=\Omega_3(0)=\eta_3(0)=1$ and parameters $K=\gamma=1$. In Fig. 1,a an output error time graphic is shown, in Fig. 1,b trajectory of "transmitted" parameter estimation $\hat{\theta}$ is presented. According to last graphic it is possible to conclude, that signal $\hat{\theta}(t)$ converges to its desired value θ.

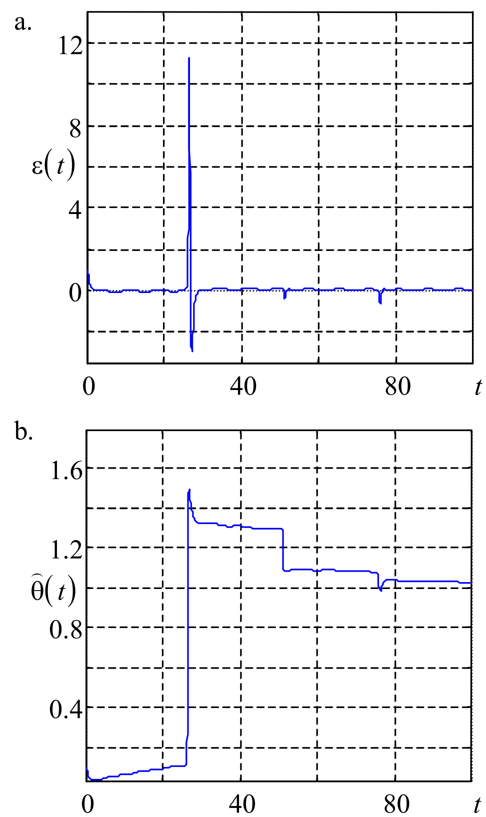

Fig. 1. Simulation result for Duffing's model.

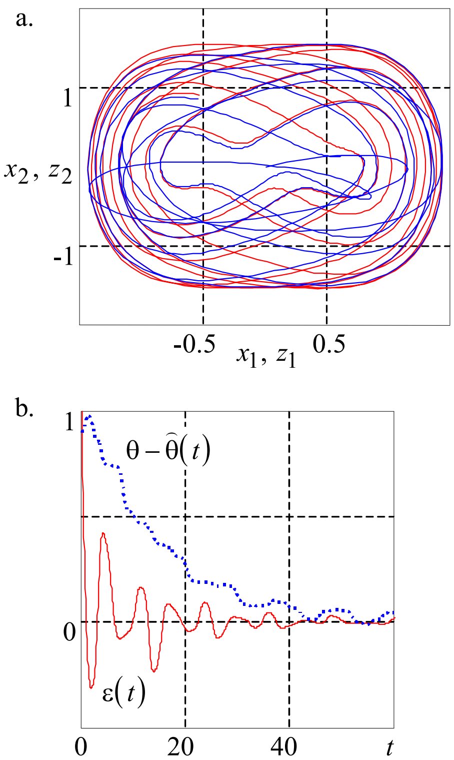

Fig. 2. Simulation result for brusselator model.

B. Forced Duffing's model

As in the previous example, to exclude explicit time dependence we should introduce auxiliary variables x_3 and x_4 in model (2):

$$\begin{aligned} &\dot{x}_1 = x_2;\ y = x_1;\\ &\dot{x}_2 = -a x_1 + \theta x_1^3 + B x_3;\\ &\dot{x}_3 = \omega x_4;\\ &\dot{x}_4 = -\omega x_3. \end{aligned} \tag{17}$$

For some bounded set of initial conditions $|x_1(0)| \le 1$ and $|x_2(0)| \le 1$ this system produces bounded solution (initial conditions for auxiliary variables $x_3(0)$ and $x_4(0)$ are chosen to guarantee desired sinusoidal signal on the input of Duffing's model). Thus, model (17) for such initial conditions satisfies to Assumption 1.

Let us consider the following observer for this model:

$$\begin{aligned} &\dot{z}_1 = z_2 + K(y - z_1);\ \hat{y} = z_1;\\ &\dot{z}_2 = -a z_1 + \hat{\theta} y^3 + B z_3;\\ &\dot{z}_3 = \omega z_4;\\ &\dot{z}_4 = -\omega z_3, \end{aligned} \tag{18}$$

The dynamics of state estimation error $\mathbf{e} = \mathbf{x} - \mathbf{z}$ take form:

$$\begin{aligned} &\dot{e}_1 = e_2 - K e_1;\ \varepsilon = e_1;\\ &\dot{e}_2 = -a e_1 + (\theta - \hat{\theta}) y^3 + B e_3;\\ &\dot{e}_3 = \omega e_4;\\ &\dot{e}_4 = -\omega e_3. \end{aligned}$$

For any $K > 0$ matrix

$$\mathbf{G}(y) = \begin{bmatrix} -K & 1 & 0 & 0\\ -a & 0 & B & 0\\ 0 & 0 & 0 & \omega\\ 0 & 0 & -\omega & 0 \end{bmatrix}$$

has two roots with strictly negative real parts and two complex conjugated roots with zero real parts, thus for any initial conditions system (4) with such matrix $\mathbf{G}$ admits conditions of Assumption 3. Assumption 2 is also satisfied for $V(e_1) = 0.5 e_1^2$, due to an upper bound for time derivative of function V can be rewritten as follows:

$$\dot{V} \le -K V + 0.5 K^{-1} e_2^2 .$$

Augmented error systems and adaptation algorithm have form:

$$\begin{aligned} &\dot{\Omega}_1 = \Omega_2 - K\Omega_1;\\ &\dot{\Omega}_2 = -a\Omega_1 + B\Omega_3 + y^3;\\ &\dot{\Omega}_3 = \omega\Omega_4;\\ &\dot{\Omega}_4 = -\omega\Omega_3; \end{aligned} \tag{19}$$

$$\begin{aligned} &\dot{\eta}_1 = \eta_2 - K\eta_1 - \Omega_1\dot{\hat{\theta}};\\ &\dot{\eta}_2 = -a\eta_1 + B\eta_3 - \Omega_2\dot{\hat{\theta}};\\ &\dot{\eta}_3 = \omega\eta_4 - \Omega_3\dot{\hat{\theta}};\\ &\dot{\eta}_4 = -\omega\eta_3 - \Omega_4\dot{\hat{\theta}}; \end{aligned} \tag{20}$$

$$\dot{\hat{\theta}} = \gamma\Omega_1(y - z_1 + \eta_1). \tag{21}$$

The result of computer simulation of system (17)–(21) is shown in Fig. 2. All initial conditions for simulations were taken zero except $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = \Omega_3(0) = \eta_3(0) = 1$ and parameters $K = \gamma = 1$. In Fig. 2, a state space trajectories are presented for transmitter model and observer, in Fig. 2,b output and parameter estimation errors time graphics are shown.

V. Conclusion

Set of applicability conditions for a scheme of adaptive observer, proposed in [8], is introduced and substantiated. These new conditions weaken requirements imposed on adaptive observer scheme in [8]. Such a weakening allows to enlarge class of admissible transmitter systems. The main advantage of these conditions consists in specializing of "estimation" goals for adaptive observer system, i.e. proposed in paper [8] requirement of output estimation error asymptotic convergence was replaced to simple boundedness.

References

[1] B. R. Andrievsky, “Information transmission by adaptive identification with chaotic carrier,“ in *Proc. 2nd Int. Conf. "Control of Oscillation and Chaos"*, St. Petersburg, 5 – 7 July, **1**, 2000, pp. 115–117.

[2] B. R. Andrievsky, “Adaptive synchronization methods for signal transmission on chaotic carriers,” *Mathem. and Computers in Simulation*, 2002.

[3] B. R. Andrievsky and A. L. Fradkov, “Information transmission by adaptive synchronisation with chaotic carrier and noisy channel,” in *Proc. 39th IEEE Conf. Decision and Control*, Sydney, 12 –15 Dec., 2000, pp. 1025–1030.

[4] A. L. Fradkov, “Adaptive synchronisation of hyper-minimum-phase systems with nonlinearities,” in *Proc. of 3rd IEEE Mediterranean Symp. on New Directions in Control*. Limassol, **1**, 1995, pp. 272–277.

[5] A. L. Fradkov and A. Yu. Markov, “Adaptive synchronisation of chaotic systems based on speed gradient method and passification,” *IEEE Trans. on Circuit and Systems*-I, **44**, 10, 1997, pp. 905–912.

[6] A. L. Fradkov, I. V. Miroshnik, and V. O. Nikiforov, *Nonlinear and adaptive control of complex systems*. Kluwer Academic Publishers, 1999, 528 p.

[7] A. L. Fradkov, H. Nijmeijer, and A. Markov, “Adaptive observer-based synchronisation for communications,” *Intern. J. of Bifurcation and Chaos*, **10**, 12, 2000, pp. 2807–2814.

[8] A. L. Fradkov, V. O. Nikiforov, and B. R. Andrievsky, “Adaptive observers for nonlinear nonpassifiable systems with application to signal transmission,” in *Proc. 41th IEEE Conf. Decision and Control*, Las Vegas, 10 –13 Dec. 2002, pp. 4706–4711.

[9] S. S. Ge and C. Wang, “Adaptive control of uncertain Chua circuits,” *IEEE Trans. on Circuit and Systems*-I, **47**, 9, 2000, pp. 1397–1402.

[10] H. Huijberts, H. Nijmeijer, and R. Willems, “System identification in communication with chaotic systems,” *IEEE Trans. on Circuit and Systems*-I, **47**, 6, 2000, pp. 800–808.

[11] *IEEE Trans. on Circuits and Systems*, **44**, 10. Special issue "Chaos control and synchronization" / Eds. M. Kennedy, M. Ogorzalek, 1997.

[12] *International Journal of Circuit Theory and Applications*, **27**,6. Special issue "Communications, information processing and control using chaos / Eds. M. Hasler, J. Vandewalle, 1999.

[13] M. P. Kennedy and G. Kolumban, “Digital communications using chaos,” in: *Controlling chaos and bifurcations in engineering systems* / Ed. G. Chen, CRC Press, 1999, pp. 477–500.

[14] R. Marino, “Adaptive observers for single output nonlinear systems,’ *IEEE Trans. Aut. Contr.*, **AC–35**, 1990, pp. 1054 – 1058.

[15] A. Yu. Markov and A. L. Fradkov, “Adaptive synchronisation of coupled chaotic systems,” *Proc. Int. Conf. "Fractals and Chaos in Chemical Engineering"*, Rome, Sept. 2–5, 1996, pp. 153–154.

[16] F. Moon, *Chaotic and Fractal Dynamics. An Introduction for Applied Scientists and Engineers*. Wiley, 1992.

[17] H. Nijmeijer and I. M. Y. Mareels, “An observer looks at synchronization,” *IEEE Trans. on Circuit and Systems*-I, **44**, 10, 1997, pp. 882–890.

[18] V.O. Nikiforov and K.V. Voronov, “Adaptive backstepping with high-order tuner,” *Automatica*, **37**, 2001, pp. 1953 – 1960.

[19] G. Nikolis and I. R. Prigogine, *Self-organization in non-equilibrium systems*. Wiley, New York, 1977.

[20] V.V. Rumyantsev and A.S. Oziraner, *Stability and stabilization of motion with respect to part of variables*. Moscow, Nauka, 1987. P. 263 (in Russian).

[21] C. Wu, Y. Yang, and L. Chua, “On adaptive synchronisation and control of nonlinear dynamical systems,” *Intern. J. of Bifurcation and Chaos*, **6**, 1996, pp. 455–471.