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THEORETICAL FOUNDATIONS AND APPLICATIONS OF MULTILEVEL DISCRETE AND DISCRETE-CONTINUAL METHODS OF LOCAL STRUCTURAL ANALYSIS

P.A. Akimov[1,2,3*], Mojtaba Aslami[1]

[1]National Research Moscow State University of Civil Engineering

Yaroslavskoe Shosse 26, Moscow, 129337, Russia

[2]Russian Academy of Architecture and Construction Sciences,

Ulitsa Bolshaya Dmitrovka 26-1, Moscow, 107031, Russia

[3]Scientific Research Center "StaDyO",

3-ya Ulitsa Yamskogo Polya 18, Moscow, 125040, Russia

*e-mail: akimov@raasn.ru

Abstract. The distinctive paper is devoted to wavelet-based discrete and discrete-continual methods of local structural analysis. Theoretical foundations of methods, corresponding program implementations, one-dimensional and two-dimensional verification samples are under consideration.

1. Introduction

As known, finite element method (FEM) is the most popular, powerful and universal method of structural analysis at the present time. Applications of FEM to static problems led to the formation of resultant system of linear algebraic equations with an immense number of unknowns [3]. Generally, this is the most time-consuming stage of the computing, especially if we take into account the limitation in the power of the contemporary software and in the performance of personal computers or even advanced supercomputers and the necessity to obtain correct and accurate solution in a reasonable time. However, practically in many cases it is impossible or unreasonable to obtain such solutions for the entire structure and due to structural or loading conditions the location and approximate dimensions of critical and most vital for designers regions of the structure can be determined. The stress-strain state in these regions is of paramount importance from the standpoint of analysis and design, and may lead to structural failure or cause impairment in structural performance [1, 2].

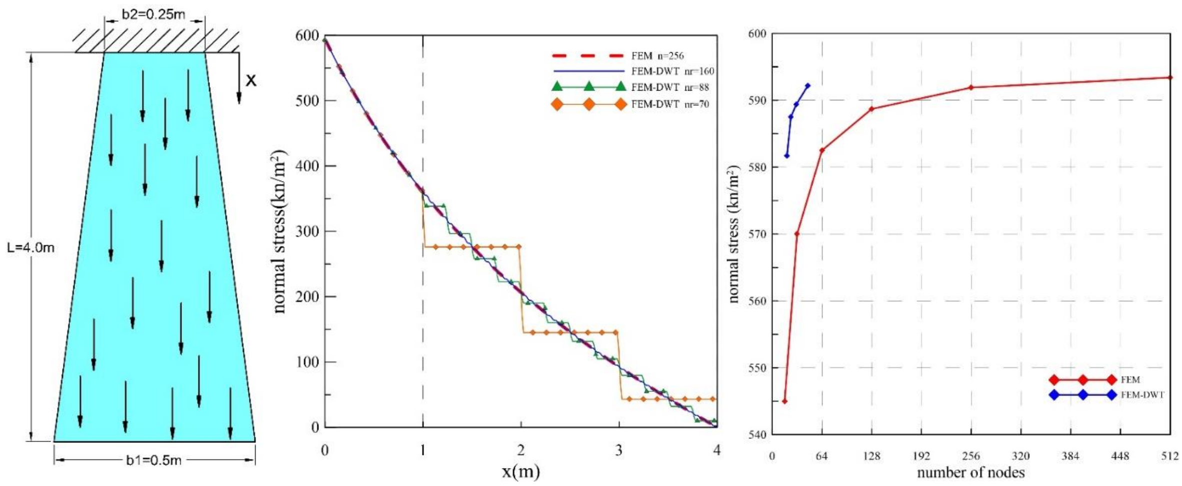
Thus, with the use of correct and appropriate method, based on multilevel structural analysis, we can achieve sufficient accuracy in definition of stress-strain state in these selected regions with a smaller number of unknowns. J. Fish et al [3] proposed a multilevel finite element approach with a superposition technique for improving the quality of numerical solutions and mathematical models of a certain class of problems. This method was presented as an attempt to construct a nearly optimal discretization scheme and improve the quality of the solution without changing the mesh size by superimposing a sequence of overlapping finite element meshes on the portion(s) of the initial finite element mesh. Another multilevel approach for structural analysis, which is applicable after FEM discretization, is a coupling of solution method with multilevel mathematical tools. Wavelet analysis is a powerful computational-analytical tool for the decomposition and multilevel mathematical modeling of functions [1, 2]. Wavelet has extraordinary characteristics and combines the advantages of functional analysis, Fourier transform, spline analysis, harmonic analysis and numerical analysis as well, and can be successfully employed for the considering goal. In this approach, after discretization and obtaining of governing equations, the considering problem is transformed into a multilevel space by multilevel wavelet transform. In recent years, P.A. Akimov et al. [1, 2] have developed this multilevel analysis approach by combining so-called discrete-continual finite element method (DCFEM) [1] and discrete wavelet transform (DWT) [2]. This method is applicable for structures, which have constant, piecewise constant or in general regular physical and geometrical parameters along one of the coordinate's directions (so-called "basic" direction). In addition, for the solution of the simplest problem of local static analysis of Bernoulli Beam on elastic foundations authors used finite difference method and discrete wavelet transform and obtained accurate results. The efficiency of the computational complexity of the proposed method can be evaluated by the comparison with the unreduced (n) and reduced (n_r) total number of degree of freedom of the structure. Let, $N_{comp} = O(n^3)$ and $N^r_{comp} = O(n_r^3)$ are the approximate computational complexity of the FEM and FEM-DWT, respectively. Then, the comparative reduction in size of the computation can be approximated by $N^r_{comp} / N_{comp} = O((n_r / n)^3)$.

2. Program Implementation

FEM-DWT method (based on coupling of FEM and DWT) and DCFEM-DWT method (based on coupling of DCFEM and DWT), considering in this paper, has been realized in software DCFEM3D. Programming environment is Microsoft Visual Studio 2013 and Intel Parallel Studio XE 2015 (Intel Visual Fortran Composer XE 2015). Corresponding software has been verified on a representative set of one-dimensional, two-dimensional and three-dimensional problems of structural analysis; several of them are presented below.

3. One-Dimensional Numerical Local Static Analysis of Thin Plate

One dimensional local solution of thin plate with uniformly varying area subject to self-weight only, are studied by the FEM-DWT method (Fig. 1a). The thickness of the plate is equal to $t = 0.02$ m. The module of elasticity of material of structure is equal to $E = 200$ GP, density is equal to $\rho = 85$ kN/m^3.



a) b) c)

Fig. 1. Thin plate subjected to self-weight: a) Initial Formulation; b) comparison of FEM and FEM-DWT results of plate analysis (normal stress); c) comparison of FEM and FEM-DWT results of plate analysis (maximum stress).

The first goal of the problem is obtain high accuracy local solution in the interval (region) $0 \le x \le 0.25L$ and the second one is compute maximum stress in the plate, by using the maximum degree of localization. Initial solution by FEM has been obtained by using 256 nodes ($M = 8$). The same discretization also used initially for local analysis. A comparison between the local solution (FEM-DWT method) and the FEM solution shows in Fig. 1b and Fig 1c. The local results obtained by FEM-DWT method agree well with the FEM results in the selected interval, even for high reduction in size of the problem. In this example the reduction has been imposed in one side of the stiffness matrix (i.e. last 0.75 part of the problem), and we obtained the maximum localization (minimum size of the problem) equal to 70 nodes with high accuracy results (Fig. 1b). Figure 2b shows the comparison of maximum stress in the plate and its relative nodes for FEM solution and FEM-DWT solution. As the results shows, with the use of FEM-DWT method, we can obtain the maximum stress in much more less number of nodes (size of governing equation) with acceptable accuracy by comparison of the high number of nodes in FEM.

4. Two-dimensional Semianalytical Local Static Analysis of Deep Beam

Deep beam, subjected to concentrated force $P = 1000$ kN, are studied by the DCFEM-DWT method (physical and geometrical parameters of structure are specified at Fig. 2). The goal of the problem is obtain high accuracy local solution in the region marked by hatching.

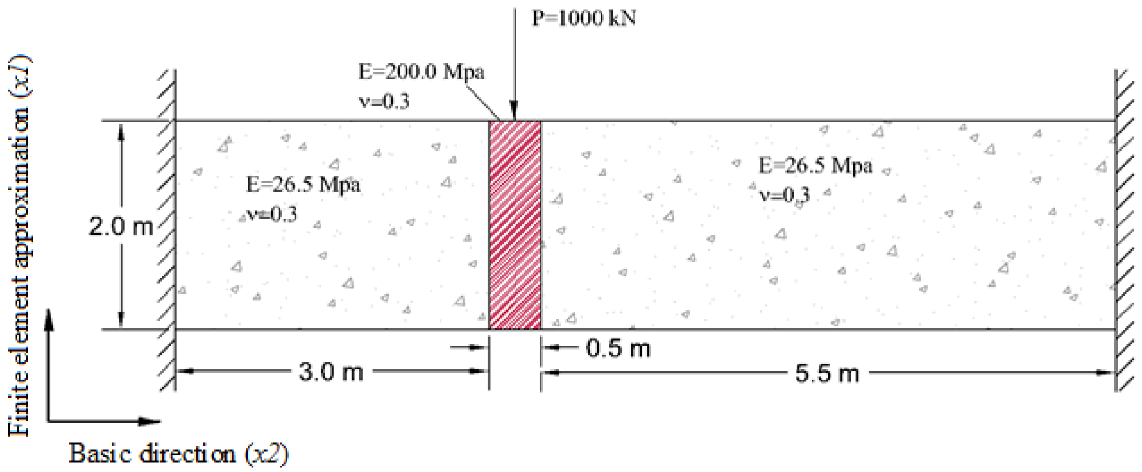


Fig. 2. Deep Beam, subjected to concentrated force.

In order to obtain FEM solution, the deep beam is discretized by 4-nodes isoparametric quadrilateral elements. We used 31 discrete-continual finite elements within DCFEM. Cross-sections $x_2 = 0.15$ m and $x_1 = 1.94$ m have been selected for comparisons of results (particularly stresses). Corresponding results agree well even with considerable reduction (Fig. 3).

5. Two-Dimensional Numerical Local Static Analysis of Deep Beam

Thin deep beam, subjected to concentrated force $P = 1000$ kN, are studied by the FEM-DWT method (Fig. 4a). The thickness of the plate is equal to unit and its module of elasticity and Poisson ratio of material is equal to $E = 20$ GP and $\nu = 0.3$, respectively. In order to obtain FEM solution, the plate is discretized by 4-nodes isoparametric quadrilateral elements. With respect to problem constrains and loading condition, the localization has been imposed in two direction. Therefore, the goal of the problem is obtain high accuracy local solution in the region $0.5 \le x < 1.0$ and $0.5 \le y < 1.0$. Initial FEM solution was obtained with $n = 256$ and then three types of reduction were imposed.

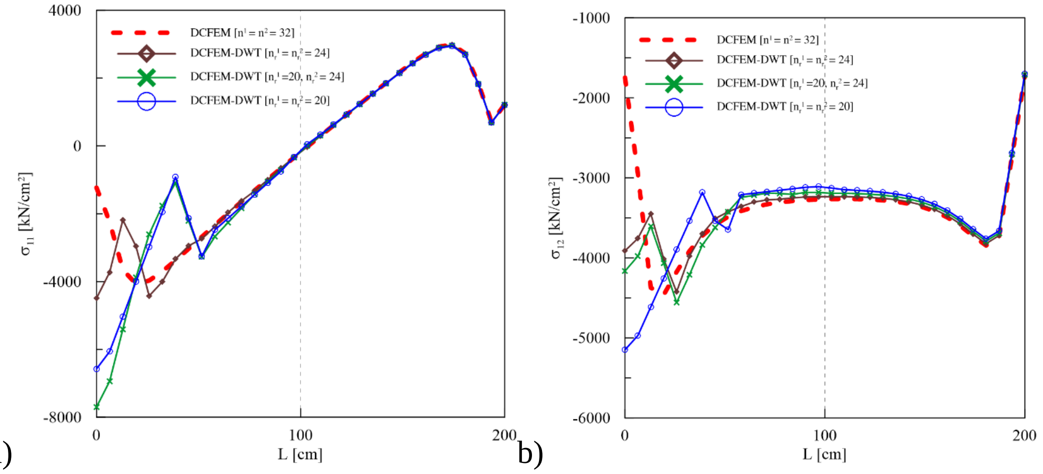


Fig. 3. Comparison of DCFEM and DCFEM-DWT results of deep beam analysis (stresses): a) σ_{11} at $x_2 = 0.15$ m; b) σ_{12} at $x_2 = 0.15$ m.

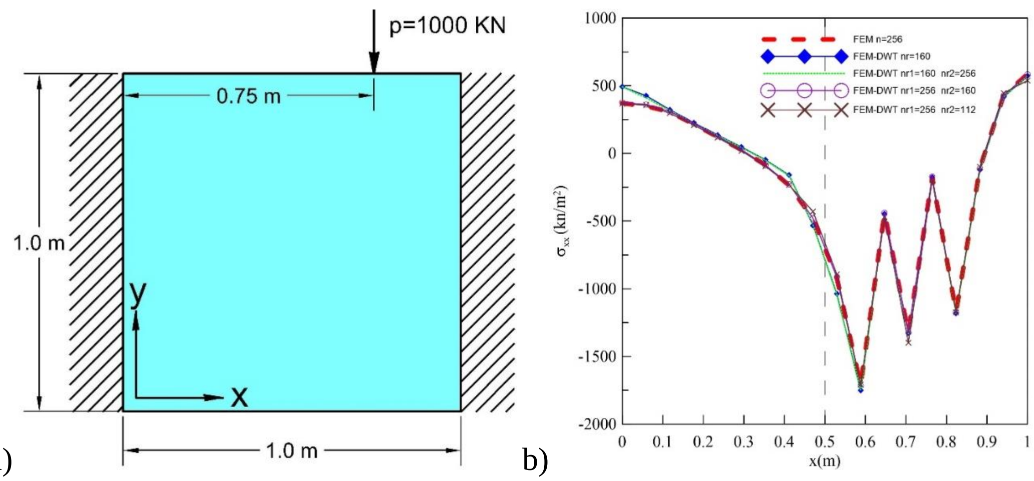


Fig. 4. Deep beam plate subjected to concentrated force: a) Initial Formulation; b) comparison of FEM and FEM-DWT results of structural analysis (σ_{xx} for $y = 0.5$ m and along the x).

First of all, localization by node was performed and the size of the problem reduced to $n_r = 160$. Then localization by degree of freedom along x ($n_{r,1}$) and y ($n_{r,2}$) directions was performed as well. Cross-section $y = 0.5$ m has been selected for comparisons of the evaluated stresses along x direction of the plate (Fig. 4b) . Corresponding results agree well. The produced turbulence near the localization line is caused by nearing reduced node.

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