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EFFECT OF NON-HOMOGENEITY IN A MAGNETO ELECTRO ELASTIC PLATE OF POLYGONAL CROSS-SECTIONS

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Abstract. The effect of non-homogeneity in a magneto electro elastic plate of polygonal cross sections is studied using the linear theory of elasticity. The wave equation of motion based on two-dimensional theory of elasticity is applied under the plane strain assumption of plate of polygonal shape, composed of non-homogeneous transversely isotropic material. The frequency equations are obtained by satisfying the irregular boundary conditions of the polygonal plate using Fourier expansion collocation method. The analytical results obtained in the physical domain have been computed numerically for Triangle, Square, Pentagon and Hexagonal plates. The results for stress, strain, displacements, induced electric and magnetic fields have been presented graphically.

Keywords: magneto-electro elastic cylinder, solid with polygonal cross sections, Fourier expansion collocation method, stresses/vibration, transducers, sensors/actuators, MEMS/NEMS

I. Introduction

The three dimensional vibration in plates of polygonal cross section made of smart and intelligent materials has considerable importance for a long time. The electro-magneto-elastic materials exhibit a desirable coupling effect between electric and magnetic fields, which are useful in smart structure applications. These materials have the capacity to react corresponding response due to the external stimulation and impulse load. The advantages of non homogeneous material still remain the structural integrity than the conventional composite materials under severe conditions. The composite consisting of piezoelectric and piezomagnetic have found increasing application in engineering structures, particularly in smart/intelligent structure system. The magneto-electro-elastic materials are used as magnetic field probes, electric packing, acoustic, hydrophones, medical, ultrasonic image processing, sensors and actuators with the responsibility of magnetic-electro-mechanical energy conversion.

Recently, many researchers have devoted their attention to the mechanics problems of transversely isotropic material connected with magneto-electro-elasticity. Ahmadi and Eskandari [1] investigated the vibration analysis of a rigid circular disk embedded in a transversely isotropic solid. Green's functions of a surface-stiffened transversely isotropic half-space were developed by Eskandari and Ahmadi [2]. Ahmadi and Eskandari [3] studied the axisymmetric circular indentation of a half-space reinforced by a buried elastic thin film. Eskandari et al. [4] analyzed the time-harmonic response of a surface stiffened transversely isotropic half-space. Weaver et al. studied the transient elastic waves in a transversely

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isotropic Plate, Haojiang et al. [5] investigated the free axisymmetric vibration of transversely isotropic piezoelectric circular plates

Pan [6] and Pan and Heyliger [7] analyzed the three-dimensional behavior of magnetoelectroelastic laminates under simple support boundary conditions. An exact solution for magnetoelectroelastic laminates in cylindrical bending has also been obtained by Pan and Heyliger [8]. Pan and Han [9] studied the exact solution for functionally graded and layered magneto-electro-elastic plates. Feng and Pan [10] discussed the dynamic fracture behavior of an internal interfacial crack between two dissimilar magneto-electro-elastic plates. Buchanan [11] developed the free vibration of an infinite magneto-electro-elastic cylinder. Dai and Wang [12, 13] have studied thermo-electro-elastic transient responses in piezoelectric hollow structures and hollow cylinder subjected to complex loadings. Annigeri et al. [14 – 15] studied respectively, the free vibration of clamped-clamped magneto-electro-elastic cylindrical shells, free vibration behavior of multiphase and layered magneto-electro-elastic beam, free vibrations of simply supported layered and multiphase magneto-electro-elastic cylindrical shells. Gao and Noda [16] presented the thermal-induced interfacial cracking of magnetoelectroelastic materials. Hon et al. [17] analyzed a point heat source on the surface of a semi-infinite transversely isotropic electro-magneto-thermo-elastic material. The dynamic response of a heat conducting solid bar of polygonal cross section subjected to moving heat source is discussed by Selvamani [18] using the Fourier expansion collocation method (FECM).The wave propagation in a magneto-thermo elastic wave in a transversely isotropic cylindrical panel using the wave propagation approach were investigated by Ponnusamy and Selvamani [19]. Recently, Selvamani and Ponnusamy [20] have studied the wave propagation in a generalized piezothermoelastic rotating bar of circular cross-section using three-dimensional linear theory of elasticity.

Bin et al. [21] analyzed the wave propagation in non-homogeneous magneto-electro-elastic plates. Chen et al. [22] worked on free vibration of non-homogeneous transversely isotropic magneto-electro-elastic plate. Chakraverty et al. [23] studied the flexural vibrations of non-homogeneous elliptic plates. Tanigawa [24] presented some basic thermoelastic problems for nonhomogeneous structural materials. Li [25] discussed the magnetoelectroelastic multi-inclusion and inhomogeneity problems and their applications in composite materials. Kong et al. [26] presented the thermo-magneto-dynamic stresses and perturbation of magnetic field vector in a non-homogeneous hollow cylinder. Ding et al. [27] and Hou et al. [28] presented an analytical solution to solve the transient responses of a special non-homogeneous pyroelectric hollow cylinder for piezothermoelastic axisymmetric plane strain dynamic problems. Ibrahim [29] provided a finite element method to solve the thermal shock problem in a non-homogeneous isotropic hollow cylinder with two relaxation times.

In this paper, the effect of magnetic field and non-homogeneity in a piezoelectric plate of polygonal cross sections is studied using the linear theory of elasticity. The frequency equations are obtained by satisfying the irregular boundary conditions of the polygonal plate using Fourier expansion collocation method. The analytical results obtained in the physical domain have been computed and the numerically analyzed results for the stress, strain, displacements and induced electric and magnetic fields have been presented graphically.

2. Formulation of the Problem

We consider a homogeneous transversely isotropic magneto-electro-elastic plate of polygonal cross-sections as shown in Fig. 1. The system displacements and stresses are defined by the cylindrical coordinates r, θ and z. The governing equations of motion of the electric and magnetic conduction in the absence of body force are taken from Selvamani and Ponnusamy [22]

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{1}{r}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{2}{r}\sigma_{r\theta}=\rho\frac{\partial^2 v}{\partial t^2}. \tag{1}$$

The equation of electric conduction is given by:

$$D_{r,r}+r^{-1}D_{,r}+r^{-1}D_{\theta,\theta}=0. \tag{2}$$

The equation of Magnetic conduction is given by:

$$B_{r,r}+r^{-1}B_{r}+r^{-1}B_{\theta,\theta}=0, \tag{3}$$

where:

$$\sigma_{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta},$$
$$\sigma_{\theta\theta}=c_{12}e_{rr}+c_{11}e_{\theta\theta}, \tag{4}$$
$$\sigma_{r\theta}=2c_{66}e_{r\theta}.$$

$$D_r=2e_{15}e_{rz}+\varepsilon_{11}E_r+m_{11}H_r,$$
$$D_\theta=2e_{15}e_{\theta z}+\varepsilon_{11}E_\theta+m_{11}H_\theta. \tag{5}$$

And:

$$B_r=2q_{15}e_{rz}+m_{11}E_r+\mu_{11}H_r,$$
$$B_\theta=2q_{15}e_{\theta z}+m_{11}E_\theta+\mu_{11}H_\theta, \tag{6}$$

where $\sigma_{rr},\sigma_{\theta\theta},\sigma_{r\theta}$ are the stress components, c_{11},c_{12} and c_{66} are elastic constants, ε_{11} are the dielectric constants, μ_{11} are the magnetic permeability coefficients, e_{31},e_{33},e_{15} are the piezoelectric material coefficients, m_{11} are the magnetoelectric material coefficients, ρ is the density of the material, D_r,D_θ are the electric displacements, B_r,B_θ are the magnetic displacements components. The strain e_{ij} are related to the displacements corresponding to the cylindrical coordinates are given by

$$e_{rr}=\frac{\partial u}{\partial r},\ e_{\theta\theta}=\frac{1}{r}\left(\frac{\partial v}{\partial\theta}+\frac{u}{r}\right),$$
$$e_{r\theta}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial r}-\frac{v}{r}\right), \tag{7}$$

where u and v are the mechanical displacements along the radial and circumferential directions.

The Electric field vector E_i is related to the electric potential E as:

$$E_r=-\frac{\partial E}{\partial r},\ E_\theta=-\frac{1}{r}\frac{\partial E}{\partial\theta}. \tag{8}$$

Similarly, the magnetic field vector H_i is related to the magnetic potential H as

$$H_r=-\frac{\partial H}{\partial r},\ H_\theta=-\frac{1}{r}\frac{\partial H}{\partial\theta}. \tag{9}$$

Substituting Eqs. (7) – (9) in Eqs. (1) – (6), we obtain the following stress displacement relations:

$$\sigma_{rr}=c_{11}\frac{\partial u}{\partial r}+c_{12}\left(\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{u}{r}\right),$$
$$\sigma_{\theta\theta}=c_{12}\frac{\partial u}{\partial r}+c_{11}\left(\frac{1}{r}\frac{\partial v}{\partial\theta}+\frac{u}{r}\right),$$

$$\sigma_{r\theta} = c_{66}\left(\frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}\right),$$

and

$$D_r = -\varepsilon_{11}\frac{\partial E}{\partial r} - m_{11}\frac{\partial H}{\partial r},$$
$$D_\theta = -\frac{\varepsilon_{11}}{r}\frac{\partial E}{\partial \theta} - \frac{m_{11}}{r}\frac{\partial H}{\partial \theta},$$
$$B_r = -m_{11}\frac{\partial E}{\partial r} - \mu_{11}\frac{\partial H}{\partial r},$$
$$B_\theta = -\frac{m_{11}}{r}\frac{\partial E}{\partial \theta} - \frac{\mu_{11}}{r}\frac{\partial H}{\partial \theta}. \tag{10}$$

The elastic constants c_{11}, c_{12}, c_{66}, magnetic permeability coefficient μ_{11}, electromagnetic material coefficient m_{11}, density ρ are characterized in terms of non-homogeneity of the material as follows:

$$c_{11} = (L+V)r^{2m}, c_{12} = Lr^{2m}, c_{66} = \frac{V}{2}r^{2m},$$
$$\mu_{11} = V' r^{2m}, m_{11} = m'_{11} r^{2m}, \tag{11}$$
$$\varepsilon_{11} = \varepsilon'_{11} r^{2m}, \rho = \rho_0 r^{2m},$$

where L, V, V' and $\rho_0, m'_{11}, \varepsilon'_{11}$ are constants of homogeneous matter and m is the rational number. Substituting Eq. (11) in Eq. (10) we obtain the stress displacement equations for nonhomogeneous medium:

$$\sigma_{rr} = r^{2m}\left[\left((L+V)\frac{\partial u}{\partial r} + L\left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right)\right)\right],$$
$$\sigma_{rr} = r^{2m}\left[\left(L\frac{\partial u}{\partial r} + (L+V)\left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right)\right)\right],$$
$$\sigma_{rr} = \frac{V}{2}r^{2m}\left[\left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}\right)\right],$$
$$D_r = r^{2m}\left(-\varepsilon_{11}'\frac{\partial E}{\partial r} - m_{11}'\frac{\partial H}{\partial r}\right), \tag{12}$$
$$D_\theta = r^{2m}\left(-\frac{\varepsilon_{11}'}{r}\frac{\partial E}{\partial \theta} - \frac{m_{11}'}{r}\frac{\partial H}{\partial \theta}\right),$$
$$B_r = r^{2m}\left(-m_{11}'\frac{\partial E}{\partial r} - \mu_{11}'\frac{\partial H}{\partial r}\right),$$
$$B_\theta = r^{2m}\left(-\frac{m_{11}'}{r}\frac{\partial E}{\partial \theta} - \frac{\mu_{11}'}{r}\frac{\partial H}{\partial \theta}\right).$$

Substituting the Eq. (12) in the Eqs. (1) - (3), we obtain the set of displacement equations

$$(L+V)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r^2}u\right) + \frac{V}{2}\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \left(\frac{2L+V}{2}\right)\frac{1}{r}\frac{\partial^2 v}{\partial r \partial \theta} - \left(\frac{2L+3V}{2}\right)\frac{1}{r^2}\frac{\partial v}{\partial \theta}$$
$$+\left(\frac{2m}{r}\right)\left((L+V)\frac{\partial u}{\partial r} + L\left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right)\right) = \rho\frac{\partial^2 u}{\partial t^2} \tag{13a}$$

$$\left(\frac{V}{2}\right)\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{1}{r^2}v\right)+\left(\frac{2L+V}{r}\right)\frac{\partial^2 u}{\partial r\partial\theta}-\left(\frac{2L+3V}{2}\right)\frac{1}{r^2}\frac{\partial u}{\partial\theta}$$
$$+\left(\frac{L+V}{r^2}\right)\frac{\partial^2 v}{\partial\theta^2}+\left(\frac{Vm}{r}\right)\left(\frac{\partial v}{\partial r}-\frac{v}{r}+\frac{1}{r}\frac{\partial u}{\partial\theta}\right)=\rho\frac{\partial^2 v}{\partial t^2} \quad (13b)$$

$$\varepsilon_{11}^{'}\left(\frac{\partial^2 E}{\partial r^2}+\frac{1}{r}\frac{\partial E}{\partial r}+\frac{1}{r^2}\frac{\partial^2 E}{\partial\theta^2}\right)+m_{11}^{'}\left(\frac{\partial^2 H}{\partial r^2}+\frac{1}{r}\frac{\partial H}{\partial r}+\frac{1}{r^2}\frac{\partial^2 H}{\partial\theta^2}\right)+\left(\frac{2m}{r}\right)\left(\varepsilon_{11}^{'}\frac{\partial E}{\partial r}+m_{11}^{'}\frac{\partial H}{\partial r}\right)=0 \quad (13c)$$

$$m_{11}^{'}\left(\frac{\partial^2 E}{\partial r^2}+\frac{1}{r}\frac{\partial E}{\partial r}+\frac{1}{r^2}\frac{\partial^2 E}{\partial\theta^2}\right)+V^{'}\left(\frac{\partial^2 H}{\partial r^2}+\frac{1}{r}\frac{\partial H}{\partial r}+\frac{1}{r^2}\frac{\partial^2 H}{\partial\theta^2}\right)+\left(\frac{2m}{r}\right)\left(m_{11}^{'}\frac{\partial E}{\partial r}+V^{'}\frac{\partial H}{\partial r}\right)=0 \quad (13d)$$

3. Solution of the problem

The Eqs. (13) is a coupled partial differential equation with three displacements and magnetic and electric conduction components. To uncouple the Eqs. (13), we seek the solution in the following form:

$$u(r,\theta)=\sum\varepsilon_n\left(r^{-1}\psi_{n,\theta}-\phi_{n,r}\right),$$
$$v(r,\theta)=\sum\varepsilon_n\left(-r^{-1}\phi_{n,\theta}-\psi_{n,r}\right),$$
$$w(r,\theta)=\sum\varepsilon_n W_{n,z}, \quad (14)$$
$$E(r,\theta)=\sum\varepsilon_n E_{n,z},$$
$$H(r,\theta)=\sum\varepsilon_n H_{n,z},$$

where $\phi_n(r,\theta)$, $\psi_n(r,\theta)$, $W_n(r,\theta)$, $E_n(r,\theta)$ and $H_n(r,\theta)$ are the displacement potentials.

Substituting the Eq. (14) in (13), we get

$$(L+V)\nabla_1^2\varphi_n+2m\left(\frac{L+V}{r}\frac{\partial\varphi_n}{\partial r}-\frac{L}{r^2}\varphi_n\right)-\rho_0\frac{\partial^2\varphi_n}{\partial t^2}=0, \quad (15a)$$

$$\varepsilon_{11}^{'}\nabla_1^2E_n+m_{11}^{'}\nabla_1^2H_n\frac{2m}{r}\left(\varepsilon_{11}^{'}\frac{\partial E_n}{\partial r}+m_{11}^{'}\frac{\partial H_n}{\partial r}\right)=0, \quad (15b)$$

$$m_{11}^{'}\nabla_1^2E_n+V^{'}\nabla_1^2H_n+\frac{2m}{r}\left(m_{11}^{'}\frac{\partial E_n}{\partial r}+V^{'}\frac{\partial H_n}{\partial r}\right)=0, \quad (15c)$$

and

$$\frac{V}{2}\nabla_1^2\psi_n+Vm\left(\frac{1}{r}\frac{\partial\psi_n}{\partial r}-\frac{\psi_n}{r}\right)=0, \quad (16)$$

where

$$\nabla_1^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}.$$

We consider the free vibration of non homogeneous polygonal cross-sectional plate and we seek the displacement function, electric and magnetic displacement function as:

$$\phi_n(r,\theta,t)=r^{-m}\overline{\phi}_n(r)\cos n\theta e^{i\omega t},$$
$$E_n(r,\theta,t)=r^{-m}E_n(r)\cos n\theta e^{i\omega t}, \quad (17)$$
$$H_n(r,\theta,t)=r^{-m}H_n(r)\cos n\theta e^{i\omega t},$$

and

$$\psi_n(r,\theta,t)=r^{-m}\psi_n(r)\cos n\theta e^{i\omega t}. \quad (18)$$

Using the Eqs. (17) and (18) in the Eqs. (15) and (16), we get

$$\varphi_n''(r)+\frac{1}{r}\varphi_n'(r)\left(\frac{\rho_0\omega^2a^2}{L+V}-\frac{\left(\left(m^2+n^2\right)2mL\right)}{L+V}\varphi_n(r)\right)=0\,, \tag{19}$$

which is reduced as

$$\varphi_n''(r)+\frac{1}{r}\varphi_n'(r)\left(\alpha^2r^2-\beta^2\right)\varphi_n(r)=0\,, \tag{20}$$

where $\alpha^2=\dfrac{\rho_0\omega^2a^2}{L+V}$, $\beta^2=\dfrac{\left(\left(m^2+n^2\right)2mL\right)}{L+V}$.

Equation (20) is Bessel equation with order β and its solution is given by:

$$\varphi_n(r)=\left(A_{1n}J_\beta(\alpha r)+A_{1n}'Y_\beta(\alpha r)\right)\cos n\theta=0\,, \tag{21}$$

where A_{1n} and A_{1n}' are arbitrary constants and $J_\beta(\alpha r)$ and $Y_\beta(\alpha r)$ are the Bessel functions of first and second kind of order β respectively.

Substituting Eq. (18) in to Eq. (16), we get

$$\psi_n''(r)+\frac{1}{r}\psi_n'(r)\left(\frac{2\rho_0\omega^2a^2}{V}-\frac{1}{r^2}\left(4m^2+4n+n^2\right)\right)\psi_n(r)=0\,, \tag{22}$$

which is reduced to

$$\psi_n''(r)+\frac{1}{r}\psi_n'(r)\left(k^2r^2-\delta^2\right)\psi_n(r)=0. \tag{23}$$

Equation (23) is Bessel equation with order δ and its solution is given by

$$\psi_n(r)=\left(A_{4n}J_\delta(\alpha r)+A_{4n}'Y_\delta(\alpha r)\right)\sin n\theta=0\,, \tag{24}$$

where A_{4n} and A_{4n}' are arbitrary constants and $J_\delta(\alpha r)$ and $Y_\delta(\alpha r)$ are the Bessel functions of first and second kind of order δ respectively.

Substituting Eq. (17) in to Eqs. (15), we get

$$\left(\varepsilon_{11}'\frac{\partial^2E_n}{\partial r^2}+\frac{\varepsilon_{11}'}{r}(2m+1)\frac{\partial E_n}{\partial r}+\frac{\varepsilon_{11}'}{r^2}\frac{\partial^2E_n}{\partial\theta^2}\right)+\left(m_{11}'\frac{\partial^2H_n}{\partial r^2}+\frac{m_{11}'}{r}(2m+1)\frac{\partial H_n}{\partial r}+\frac{\varepsilon_{11}'}{r^2}\frac{\partial^2H_n}{\partial\theta^2}\right)=0, \tag{25}$$

$$m_{11}'\left(E_n''(r)+\frac{1}{r}E_n'(r)-\frac{\left(m^2+n^2\right)}{r^2}E_n(r)\right)+V'\left(H_n''(r)+\frac{1}{r}H_n'(r)-\frac{\left(m^2+n^2\right)}{r^2}H_n(r)\right)=0\,, \tag{26}$$

which will reduced in to the convenient form:

$$\varepsilon_{11}'\left(E_n''(r)+\frac{1}{r}E_n'(r)-\frac{p^2}{r^2}E_n(r)\right)+m_{11}'\left(H_n''(r)+\frac{1}{r}H_n'(r)-\frac{p^2}{r^2}H_n(r)\right)=0\,, \tag{27}$$

$$m_{11}'\left(E_n''(r)+\frac{1}{r}E_n'(r)-\frac{p^2}{r^2}E_n(r)\right)+V'\left(H_n''(r)+\frac{1}{r}H_n'(r)-\frac{p^2}{r^2}H_n(r)\right)=0\,, \tag{28}$$

where

$p^2=m^2+n^2.$

Solving Eq. (27) and Eq. (28), we can get

$$E_n''(r)+\frac{1}{r}E_n'(r)-\frac{p^2}{r^2}E_n(r)=0\,, \tag{29}$$

$$H_n''(r)+\frac{1}{r}H_n'(r)-\frac{p^2}{r^2}H_n(r)=0. \tag{30}$$

The general solution of Eqs. (29) and (30) are as follows:

$$E_n(r,\theta,t) = \left(A_{2n} r^p + A'_{2n} r^{-p}\right)\cos n\theta e^{iwt}, \tag{31}$$

$$H_n(r,\theta,t) = \left(A_{3n} r^p + A'_{3n} r^{-p}\right)\cos n\theta e^{iwt}, \tag{32}$$

where $A_{2n}, A'_{2n}, A_{3n}, A'_{3n}$ are the arbitrary constants.

The general solution of the non-homogeneous solid plate of polygonal cross sections is as:

$$\phi_n(r,\theta,t) = A_{1n} J_\beta(\alpha r)\cos n\theta, \tag{33a}$$

$$E_n(r,\theta,t) = A_{2n} r^p \cos n\theta, \tag{33b}$$

$$H_n(r,\theta,t) = A_{3n} r^p \cos n\theta, \tag{33c}$$

$$\psi_n(r,\theta,t) = A_{1n} J_\delta(kr)\sin n\theta. \tag{33d}$$

4. Boundary condition and frequency equations

In this problem, the vibration of polygonal cross-sectional plate is considered. Since the boundary is irregular in shape, it is difficult to satisfy the boundary conditions along the surface of the plate directly. Hence, the Fourier expansion collocation method is applied to satisfy the boundary conditions. For the plate, the normal stress σ_{xx} and shearing stresses σ_{xy}, σ_{xz}, the electric field D_r and the magnetic field B_r is equal to zero for stress free boundary, and for rigidly fixed boundary, the displacements along the radial direction u_r, along the circumferential direction u_θ, and the electric field E, and the magnetic field H is equal to zero. Thus the following types of boundary conditions are assumed for the plate of polygonal cross-section is

(i) Stress free(unclamped edge), which leads to

$$\left(\sigma_{xx}\right)_i = \left(\sigma_{xy}\right)_i = \left(\sigma_{xz}\right)_i = \left(D_r\right)_i = \left(B_r\right)_i = 0\,; \tag{34}$$

(ii) Rigidly fixed(clamped edge), implies that

$$\left(u_r\right)_i = \left(u_\theta\right)_i = \left(E\right)_i = \left(H\right)_i = 0, \tag{35}$$

where σ_{xx} is the normal stress, σ_{xy}, σ_{xz} are the shearing stresses, D_r is the electric field, B_r is the magnetic field and the bracket $(\ )_i$ is the value at the boundary Γ_i. Similarly u_r, u_θ are displacements along the radial and circumferential direction, E and H are respectively the electric and magnetic displacements in the i^{th} segment of the polygonal cross-sectional plate. Since the vibration displacements are expressed in terms of the coordinates r and θ, it is convenient to treat the boundary conditions when the derivatives in the equations of the stresses are transformed in terms of the coordinates r and θ instead of the coordinates x_i and y_i. The relations between the displacements are as follows for i^{th} segment of straight-line boundaries

$$\begin{aligned} u_x &= u_r \cos(\theta - \gamma_i) - u_\theta \sin(\theta - \gamma_i), \\ u_y &= u_\theta \cos(\theta - \gamma_i) - u_r \sin(\theta - \gamma_i). \end{aligned} \tag{36}$$

Since the angle γ_i between the reference axis and normal of the i^{th} boundary has a constant value in a segment Γ_i, we obtain:

$$\frac{\partial r}{\partial x_i}=\cos(\theta-\gamma_i),\ \frac{\partial \theta}{\partial x_i}=-\left(\frac{1}{r}\right)\sin(\theta-\gamma_i),$$
$$\frac{\partial r}{\partial y_i}=\sin(\theta-\gamma_i),\ \frac{\partial \theta}{\partial y_i}=\left(\frac{1}{r}\right)\cos(\theta-\gamma_i). \tag{37}$$

Using the Eqs. (36) and (37), the normal and shearing stresses are transformed as:

$$\sigma_{xx}=c_{11}\cos^2(\theta-\gamma_i)+c_{12}\sin^2(\theta-\gamma_i)u_{,r}+r^{-1}\left(c_{11}\sin^2(\theta-\gamma_i)+c_{12}\cos^2(\theta-\gamma_i)\right)\left(u+v_{,\theta}\right)$$
$$+c_{66}\left(r^{-1}\left(v-u_{,\theta}\right)-v_{,r}\right)\sin 2(\theta-\gamma_i)+c_{13}w_{,z}+e_{31}E_{,zz}+q_{31}H_{,zz}=0$$
$$\sigma_{xy}=c_{66}\left(\left(u_{,r}-r^{-1}\left(v_{,\theta}+u\right)\right)\sin 2(\theta-\gamma_i)+\left(r^{-1}\left(u_{,\theta}-v\right)+v_{,r}\right)\cos 2(\theta-\gamma_i)\right)=0$$
$$\sigma_{xz}=c_{44}\left(\left(u_{,z}+w_{,r}\right)\cos(\theta-\gamma_i)-\left(v_{,z}+r^{-1}w_{,\theta}\right)\sin(\theta-\gamma_i)\right)+e_{15}\left(E_{,r}\cos(\theta-\gamma_i)-r^{-1}E_{,\theta}\sin(\theta-\gamma_i)\right)$$
$$+q_{15}\left(H_{,r}\cos(\theta-\gamma_i)-r^{-1}H_{,\theta}\sin(\theta-\gamma_i)\right)=0$$
$$D_x=-\varepsilon_{11}\frac{\partial E}{\partial r}-m_{11}\frac{\partial H}{\partial r}=0\ ,\ B_x=-m_{11}{}'\frac{\partial E}{\partial r}-\mu_{11}{}'\frac{\partial H}{\partial r}=0. \tag{38}$$

Imposing non-homogeneity to the Eq. (38), we can get the following mechanical, magnetic and electric stress equations :

$$\sigma_{xx}=(L+V)\cos^2(\theta-\gamma_i)+L\sin^2(\theta-\gamma_i)u_{,r}+r^{-1}\left((L+V)\sin^2(\theta-\gamma_i)+L\cos^2(\theta-\gamma_i)\right)\left(u+v_{,\theta}\right)$$
$$+c_{66}\left(r^{-1}\left(v-u_{,\theta}\right)-v_{,r}\right)\sin 2(\theta-\gamma_i)+c_{13}w_{,z}+e_{31}E_{,zz}+q_{31}H_{,zz}=0$$
$$\sigma_{xy}=\frac{V}{2}\left(\left(u_{,r}-r^{-1}\left(v_{,\theta}+u\right)\right)\sin 2(\theta-\gamma_i)+\left(r^{-1}\left(u_{,\theta}-v\right)+v_{,r}\right)\cos 2(\theta-\gamma_i)\right)=0$$
$$D_x=-\varepsilon_{11}\frac{\partial E}{\partial r}-m_{11}\frac{\partial H}{\partial r}=0$$
$$B_x=-m_{11}\frac{\partial E}{\partial r}-\mu_{11}\frac{\partial H}{\partial r}=0. \tag{39}$$

Substituting the Eqs. (33a) - (33d) in the Eq. (34), the boundary conditions are transformed for stress free non-homogeneous polygonal cross-sectional plate as follows:

$$\left[\left(S_{xx}\right)_i+\left(\bar{S}_{xx}\right)_i\right]e^{i\omega t}$$
$$\left[\left(S_{xy}\right)_i+\left(\bar{S}_{xy}\right)_i\right]e^{i\omega t}$$
$$\left[\left(E_x\right)_i+\left(\bar{E}_x\right)_i\right]e^{i\omega t}$$
$$\left[\left(H_x\right)_i+\left(\bar{H}_x\right)_i\right]e^{i\omega t},$$

where

$$S_{xx}=0.5\left(A_{10}e_0^1+A_{20}e_0^2+A_{30}e_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}e_n^1+A_{2n}e_n^2+A_{30}e_n^3+A_{4n}e_n^4\right)$$
$$S_{xy}=0.5\left(A_{10}f_0^1+A_{20}f_0^2+A_{30}f_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}f_n^1+A_{2n}f_n^2+A_{30}f_n^3+A_{4n}f_n^4\right)$$
$$S_{xz}=0.5\left(A_{10}g_0^1+A_{20}g_0^2+A_{30}g_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}g_n^1+A_{2n}g_n^2+A_{30}g_n^3+A_{4n}g_n^4\right)$$
$$E=0.5\left(A_{10}h_0^1+A_{20}h_0^2+A_{30}h_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}h_n^1+A_{2n}h_n^2+A_{30}h_n^3+A_{4n}h_n^4\right)$$

$$H = 0.5\left(A_{10}i_0^1 + A_{20}i_0^2 + A_{30}i_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n}i_n^1 + A_{2n}i_n^2 + A_{30}i_n^3 + A_{4n}i_n^4\right) \tag{40}$$

$$\bar{S}_{xx} = 0.5\bar{e}_0^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(A_{1n}\bar{e}_n^1 + A_{2n}\bar{e}_n^2 + A_{30}\bar{e}_n^3 + A_{4n}\bar{e}_n^4\right)$$

$$\bar{S}_{xy} = 0.5\bar{f}_0^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(A_{1n}\bar{f}_n^1 + A_{2n}\bar{f}_n^2 + A_{30}\bar{f}_n^3 + A_{4n}\bar{f}_n^4\right)$$

$$\bar{S}_{xz} = 0.5\bar{g}_0^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(A_{1n}\bar{g}_n^1 + A_{2n}\bar{g}_n^2 + A_{30}\bar{g}_n^3 + A_{4n}\bar{g}_n^4\right)$$

$$\bar{E} = 0.5\bar{h}_0^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(A_{1n}\bar{h}_n^1 + A_{2n}\bar{h}_n^2 + A_{30}\bar{h}_n^3 + A_{4n}\bar{h}_n^4\right)$$

$$\bar{H} = 0.5\bar{i}_0^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(A_{1n}\bar{i}_n^1 + A_{2n}\bar{i}_n^2 + A_{30}\bar{i}_n^3 + A_{4n}\bar{i}_n^4\right). \tag{41}$$

The coefficients $e_n^i \sim \bar{i}_n^i$ are given in the Appendix A.

Performing the Fourier series expansion to Eq. (35) along the boundary, the boundary conditions along the surface are expanded in the form of double Fourier series. In the symmetric mode, the boundary conditions are obtained as:

$$\sum_{m=0}^{\infty}\varepsilon_m\left[E_{m0}^1A_{10} + E_{m0}^2A_{20} + E_{m0}^3A_{30} + E_{m0}^4A_{40} + \sum_{n=1}^{\infty}\left(E_{mn}^1A_{1n} + E_{mn}^2A_{2n} + E_{mn}^3A_{3n} + E_{mn}^4A_{4n} + E_{mn}^5A_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\varepsilon_m\left[F_{m0}^1A_{10} + F_{m0}^2A_{20} + F_{m0}^3A_{30} + F_{m0}^4A_{40} + \sum_{n=1}^{\infty}\left(F_{mn}^1A_{1n} + F_{mn}^2A_{2n} + F_{mn}^3A_{3n} + F_{mn}^4A_{4n} + F_{mn}^5A_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\varepsilon_m\left[G_{m0}^1A_{10} + G_{m0}^2A_{20} + G_{m0}^3A_{30} + G_{m0}^4A_{40} + \sum_{n=1}^{\infty}\left(G_{mn}^1A_{1n} + G_{mn}^2A_{2n} + G_{mn}^3A_{3n} + G_{mn}^4A_{4n} + G_{mn}^5A_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\varepsilon_m\left[H_{m0}^1A_{10} + H_{m0}^2A_{20} + H_{m0}^3A_{30} + H_{m0}^4A_{40} + \sum_{n=1}^{\infty}\left(H_{mn}^1A_{1n} + H_{mn}^2A_{2n} + H_{mn}^3A_{3n} + H_{mn}^4A_{4n} + H_{mn}^5A_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\varepsilon_m\left[I_{m0}^1A_{10} + I_{m0}^2A_{20} + I_{m0}^3A_{30} + I_{m0}^4A_{40} + \sum_{n=1}^{\infty}\left(I_{mn}^1A_{1n} + I_{mn}^2A_{2n} + I_{mn}^3A_{3n} + I_{mn}^4A_{4n} + I_{mn}^5A_{5n}\right)\right] = 0\,. \tag{42}$$

Similarly, for the antisymmetric mode, the boundary conditions are expressed as:

$$\sum_{m=0}^{\infty}\left[\bar{E}_{m0}^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(\bar{E}_{mn}^1\bar{A}_{1n} + \bar{E}_{mn}^2\bar{A}_{2n} + \bar{E}_{mn}^3\bar{A}_{3n} + \bar{E}_{mn}^4\bar{A}_{4n} + \bar{E}_{mn}^5\bar{A}_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\left[\bar{F}_{m0}^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(\bar{F}_{mn}^1\bar{A}_{1n} + \bar{F}_{mn}^2\bar{A}_{2n} + \bar{F}_{mn}^3\bar{A}_{3n} + \bar{F}_{mn}^4\bar{A}_{4n} + \bar{F}_{mn}^5\bar{A}_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\left[\bar{G}_{m0}^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(\bar{G}_{mn}^1\bar{A}_{1n} + \bar{G}_{mn}^2\bar{A}_{2n} + \bar{G}_{mn}^3\bar{A}_{3n} + \bar{G}_{mn}^4\bar{A}_{4n} + \bar{G}_{mn}^5\bar{A}_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\left[\bar{H}_{m0}^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(\bar{H}_{mn}^1\bar{A}_{1n} + \bar{H}_{mn}^2\bar{A}_{2n} + \bar{H}_{mn}^3\bar{A}_{3n} + \bar{H}_{mn}^4\bar{A}_{4n} + \bar{H}_{mn}^5\bar{A}_{5n}\right)\right] = 0$$

$$\sum_{m=0}^{\infty}\left[\bar{I}_{m0}^5\bar{A}_{50} + \sum_{n=1}^{\infty}\left(\bar{I}_{mn}^1\bar{A}_{1n} + \bar{I}_{mn}^2\bar{A}_{2n} + \bar{I}_{mn}^3\bar{A}_{3n} + \bar{I}_{mn}^4\bar{A}_{4n} + \bar{I}_{mn}^5\bar{A}_{5n}\right)\right] = 0\,, \tag{43}$$

where

$$E_{mn}^j = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} e_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$

$$F_{mn}^j = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} f_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$

$$G_{mn}^{j} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} g_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$

$$H_{mn}^{j} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} h_n^j\left(R_i,\theta\right)\cos m\theta d\theta \tag{44}$$

$$\overline{E}_{mn}^{j} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{e}_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$

$$\overline{F}_{mn}^{j} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{f}_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$

$$\overline{G}_{mn}^{j} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{g}_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$

$$\overline{H}_{mn}^{j} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{h}_n^j\left(R_i,\theta\right)\sin m\theta d\theta, \tag{45}$$

where j=1,2,3,4, I is the number of segments, R_i is the coordinate r at the boundary and N is the number of truncation of the Fourier series. The frequency equations are obtained by truncating the series to N+1 terms, and equating the determinant of the coefficients of the amplitude $A_{in} = 0$ and $\overline{A}_{in} = 0$ (i=1,2,3,4), for symmetric and anti symmetric modes of vibrations. When the plate is symmetric about more than one axis, the boundary conditions in the case of symmetric mode can be written in the form of matrix as given below:

$$\begin{bmatrix}
E_{00}^1 & E_{00}^2 & E_{00}^3 & E_{00}^4 & 0 & E_{01}^1 & \cdots & E_{0N}^1 & E_{01}^2 & \cdots & E_{0N}^2 & E_{01}^3 & \cdots & E_{0N}^3 & E_{01}^4 & \cdots & E_{0N}^4 & E_{01}^5 & \cdots & E_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
E_{N0}^1 & E_{N0}^2 & E_{N0}^3 & E_{N0}^4 & 0 & E_{N1}^1 & \cdots & E_{NN}^1 & E_{N1}^2 & \cdots & E_{NN}^2 & E_{N1}^3 & \cdots & E_{NN}^3 & E_{N1}^4 & \cdots & E_{NN}^4 & E_{N1}^5 & \cdots & E_{NN}^5 \\
F_{00}^1 & F_{00}^2 & F_{00}^3 & F_{00}^4 & 0 & F_{01}^1 & \cdots & F_{0N}^1 & F_{01}^2 & \cdots & F_{0N}^2 & F_{01}^3 & \cdots & F_{0N}^3 & F_{01}^4 & \cdots & F_{0N}^4 & F_{01}^5 & \cdots & F_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
F_{N0}^1 & F_{N0}^2 & F_{N0}^3 & F_{N0}^4 & 0 & F_{N1}^1 & \cdots & F_{NN}^1 & F_{N1}^2 & \cdots & F_{NN}^2 & F_{N1}^3 & \cdots & F_{NN}^3 & F_{N1}^4 & \cdots & F_{NN}^4 & F_{N1}^5 & \cdots & F_{NN}^5 \\
G_{00}^1 & G_{00}^2 & G_{00}^3 & G_{00}^4 & 0 & G_{01}^1 & \cdots & G_{0N}^1 & G_{01}^2 & \cdots & G_{0N}^2 & G_{01}^3 & \cdots & G_{0N}^3 & G_{01}^4 & \cdots & G_{0N}^4 & G_{01}^5 & \cdots & G_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
G_{N0}^1 & G_{N0}^2 & G_{N0}^3 & G_{N0}^4 & 0 & G_{N1}^1 & \cdots & G_{NN}^1 & G_{N1}^2 & \cdots & G_{NN}^2 & G_{N1}^3 & \cdots & G_{NN}^3 & G_{N1}^4 & \cdots & G_{NN}^4 & G_{N1}^5 & \cdots & G_{NN}^5 \\
H_{00}^1 & H_{00}^2 & H_{00}^3 & H_{00}^4 & 0 & H_{01}^1 & \cdots & H_{0N}^1 & H_{01}^2 & \cdots & H_{0N}^2 & H_{01}^3 & \cdots & H_{0N}^3 & H_{01}^4 & \cdots & H_{0N}^4 & H_{01}^5 & \cdots & H_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
H_{N0}^1 & H_{N0}^2 & H_{N0}^3 & H_{N0}^4 & 0 & H_{N1}^1 & \cdots & H_{NN}^1 & H_{N1}^2 & \cdots & H_{NN}^2 & H_{N1}^3 & \cdots & H_{NN}^3 & H_{N1}^4 & \cdots & H_{NN}^4 & H_{N1}^5 & \cdots & H_{NN}^5 \\
I_{00}^1 & I_{00}^2 & I_{00}^3 & I_{00}^4 & 0 & I_{01}^1 & \cdots & I_{0N}^1 & I_{01}^2 & \cdots & I_{0N}^2 & I_{01}^3 & \cdots & I_{0N}^3 & I_{01}^4 & \cdots & I_{0N}^4 & I_{01}^5 & \cdots & I_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
I_{N0}^1 & I_{N0}^2 & I_{N0}^3 & I_{N0}^4 & 0 & I_{N1}^1 & \cdots & I_{NN}^1 & I_{N1}^2 & \cdots & I_{NN}^2 & I_{N1}^3 & \cdots & I_{NN}^3 & I_{N1}^4 & \cdots & I_{NN}^4 & I_{N1}^5 & \cdots & I_{NN}^5
\end{bmatrix}
\begin{bmatrix} A_{10} \\ A_{20} \\ A_{30} \\ A_{40} \\ A_{50} \\ A_{11} \\ \vdots \\ A_{1N} \\ A_{21} \\ \vdots \\ A_{2N} \\ \vdots \\ A_{51} \\ \vdots \\ A_{5N} \end{bmatrix} = 0 \tag{46}$$

Similarly, the matrix for the antisymmetric mode is obtained as:

$$\begin{bmatrix}
\overline{E}_{10}^{5} & \overline{E}_{11}^{1} & \cdots & \overline{E}_{1N}^{1} & \overline{E}_{11}^{2} & \cdots & \overline{E}_{1N}^{2} & \overline{E}_{11}^{3} & \cdots & \overline{E}_{1N}^{3} & \overline{E}_{11}^{4} & \cdots & \overline{E}_{1N}^{4} & \overline{E}_{11}^{5} & \cdots & \overline{E}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{E}_{N0}^{5} & \overline{E}_{N1}^{1} & \cdots & \overline{E}_{NN}^{1} & \overline{E}_{N1}^{2} & \cdots & \overline{E}_{NN}^{2} & \overline{E}_{N1}^{3} & \cdots & \overline{E}_{NN}^{3} & \overline{E}_{N1}^{4} & \cdots & \overline{E}_{NN}^{4} & \overline{E}_{N1}^{5} & \cdots & \overline{E}_{NN}^{5} \\
\overline{F}_{10}^{5} & \overline{F}_{11}^{1} & \cdots & \overline{F}_{1N}^{1} & \overline{F}_{11}^{2} & \cdots & \overline{F}_{1N}^{2} & \overline{F}_{11}^{3} & \cdots & \overline{F}_{1N}^{3} & \overline{F}_{11}^{4} & \cdots & \overline{F}_{1N}^{4} & \overline{F}_{11}^{5} & \cdots & \overline{F}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{F}_{N0}^{5} & \overline{F}_{N1}^{1} & \cdots & \overline{F}_{NN}^{1} & \overline{F}_{N1}^{2} & \cdots & \overline{F}_{NN}^{2} & \overline{F}_{N1}^{3} & \cdots & \overline{F}_{NN}^{3} & \overline{F}_{N1}^{4} & \cdots & \overline{F}_{NN}^{4} & \overline{F}_{N1}^{5} & \cdots & \overline{F}_{NN}^{5} \\
\overline{G}_{10}^{5} & \overline{G}_{11}^{1} & \cdots & \overline{G}_{1N}^{1} & \overline{G}_{11}^{2} & \cdots & \overline{G}_{1N}^{2} & \overline{G}_{11}^{3} & \cdots & \overline{G}_{1N}^{3} & \overline{G}_{11}^{4} & \cdots & \overline{G}_{1N}^{4} & \overline{G}_{11}^{5} & \cdots & \overline{G}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{G}_{N0}^{5} & \overline{G}_{N1}^{1} & \cdots & \overline{G}_{NN}^{1} & \overline{G}_{N1}^{2} & \cdots & \overline{G}_{NN}^{2} & \overline{G}_{N1}^{3} & \cdots & \overline{G}_{NN}^{3} & \overline{G}_{N1}^{4} & \cdots & \overline{G}_{NN}^{4} & \overline{G}_{N1}^{5} & \cdots & \overline{G}_{NN}^{5} \\
\overline{H}_{10}^{5} & \overline{H}_{11}^{1} & \cdots & \overline{H}_{1N}^{1} & \overline{H}_{11}^{2} & \cdots & \overline{H}_{1N}^{2} & \overline{H}_{11}^{3} & \cdots & \overline{H}_{1N}^{3} & \overline{H}_{11}^{4} & \cdots & \overline{H}_{1N}^{4} & \overline{H}_{11}^{5} & \cdots & \overline{H}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{H}_{N0}^{5} & \overline{H}_{N1}^{1} & \cdots & \overline{H}_{NN}^{1} & \overline{H}_{N1}^{2} & \cdots & \overline{H}_{NN}^{2} & \overline{H}_{N1}^{3} & \cdots & \overline{H}_{NN}^{3} & \overline{H}_{N1}^{4} & \cdots & \overline{H}_{NN}^{4} & \overline{H}_{N1}^{5} & \cdots & \overline{H}_{NN}^{5} \\
\overline{I}_{10}^{5} & \overline{I}_{11}^{1} & \cdots & \overline{I}_{1N}^{1} & \overline{I}_{11}^{2} & \cdots & \overline{I}_{1N}^{2} & \overline{I}_{11}^{3} & \cdots & \overline{I}_{1N}^{3} & \overline{I}_{11}^{4} & \cdots & \overline{I}_{1N}^{4} & \overline{I}_{11}^{5} & \cdots & \overline{I}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{I}_{N0}^{5} & \overline{I}_{N1}^{1} & \cdots & \overline{I}_{NN}^{1} & \overline{I}_{N1}^{2} & \cdots & \overline{I}_{NN}^{2} & \overline{I}_{N1}^{3} & \cdots & \overline{I}_{NN}^{3} & \overline{I}_{N1}^{4} & \cdots & \overline{I}_{NN}^{4} & \overline{I}_{N1}^{5} & \cdots & \overline{I}_{NN}^{5}
\end{bmatrix}
\begin{bmatrix} A_{50} \\ A_{11} \\ \vdots \\ A_{1N} \\ A_{21} \\ \vdots \\ A_{2N} \\ A_{31} \\ \vdots \\ A_{3N} \\ A_{41} \\ \vdots \\ A_{4N} \\ A_{51} \\ \vdots \\ A_{5N} \end{bmatrix} = 0 \tag{47}$$

5. Homogeneous electro-elastic plate of polygonal cross-sections

The result for homogeneous transversely isotropic electro-elastic plate of polygonal cross-sections can be obtained by omitting the magnetic conductions $B_i = 0$ (i= r, θ ,z) in the corresponding relations and expressions. Thus the displacement potential for this problem is obtained by setting piezomagnetic material coefficients $q_{15} = q_{31} = q_{33} = 0$, magnetic material coefficients $m_{11} = m_{33} = 0$ and the magnetic permeability coefficients $\mu_{11} = \mu_{33} = 0$. Therefore the Eqs. (13a)- (13d) are reduced to

$$\begin{aligned}
&\left(\bar{c}_{11}\nabla_2^2 - t_L^2 + \Omega^2\right)\bar{\phi}_n + \left(1+\bar{c}_{13}\right)t_L^2\overline{W}_n + \left(\bar{e}_{31}+\bar{e}_{15}\right)t_L^2\overline{E}_n = 0 \\
&\left(\nabla_2^2 + \Omega^2 - \bar{c}_{33}t_L^2\right)\overline{W}_n + \left(1+\bar{c}_{13}\right)\nabla_2^2\bar{\phi}_n + \left(\bar{e}_{15}\nabla_2^2 - t_L^2\right)\overline{E}_n = 0 \\
&\left(\bar{e}_{15}\nabla_2^2 - t_L^2\right)\overline{W}_n - \left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla_2^2\bar{\phi}_n + \left(\bar{\varepsilon}_{33}t_L^2 - \bar{\varepsilon}_{11}\nabla_2^2\right)\overline{E}_n = 0
\end{aligned} \tag{48}$$

and $$\left(\nabla_2^2 + \frac{\Omega^2 - t_L^2}{\bar{c}_{66}}\right)\overline{\psi}_n = 0. \tag{49}$$

Solving the Eq. (48), we obtain a trivial solution. To obtain the non-trivial solutions, put the determinant of the coefficient of the matrix is equal to zero. Thus we get

$$\begin{vmatrix}
\left(\bar{c}_{11}\nabla_2^2 + \Omega^2 - t_L^2\right) & \left(1+\bar{c}_{13}\right)t_L^2 & \left(\bar{e}_{31}+\bar{e}_{15}\right)t_L^2 \\
\left(1+\bar{c}_{13}\right)\nabla_2^2 & \left(\nabla_2^2 + \Omega^2 - \bar{c}_{33}t_L^2\right) & \left(\bar{e}_{15}\nabla_2^2 - t_L^2\right) \\
-\left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla_2^2 & \left(\bar{e}_{15}\nabla_2^2 - t_L^2\right) & \left(\bar{\varepsilon}_{33}t_L^2 - \bar{\varepsilon}_{11}\nabla_2^2\right)
\end{vmatrix}\left(\bar{\phi}_n, \overline{W}_n, \overline{E}_n\right) = 0 \tag{50}$$

Simplifying the Eq. (50), we get a six order partial differential equation, that is

$$\left(P\nabla_2^6 + Q\nabla_2^4 + R\nabla_2^2 + S\right)\phi_n = 0, \tag{51}$$

where

$$P = -\bar{c}_{11}\left(\bar{\varepsilon}_{11} + \bar{e}_{15}^2\right)$$

$$Q = \bar{c}_{11}\left(g_7 - g_5\bar{\varepsilon}_{11} + 2g_6\bar{e}_{15}\right) - g_1\left(\bar{\varepsilon}_{11} + \bar{e}_{15}^2\right) + t_L^2\left(-g_2\left(g_2\bar{\varepsilon}_{11} + 2g_3\bar{e}_{15}\right) + g_3^2\right)$$

$$R=\bar{c}_{11}\left(g_5g_7-g_6^2\right)+g_1\left(g_7-g_5\bar{\varepsilon}_{11}+2g_6\bar{e}_{15}\right)+t_L^2\left(g_2\left(g_2g_7+2g_3g_6\right)+g_3^2g_5\right)$$

$$S=g_1\left(g_5g_7-g_6^2\right). \tag{52}$$

Solving the Eq. (51), the solution for the symmetric mode is obtained as

$$\bar{\phi}_n\left(r\right)=\sum_{i=1}^{3}A_{in}J_n\left(\alpha_ir\right)\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t}$$

$$\bar{W}_n\left(r\right)=\sum_{i=1}^{3}d_iA_{in}J_n\left(\alpha_ir\right)\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t} \tag{53}$$

$$\bar{E}_n\left(r\right)=\sum_{i=1}^{3}e_iA_{in}J_n\left(\alpha_ir\right)\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t}.$$

The constants d_i and e_i defined in the Eq. (53) are given by

$$d_i=\frac{\left(\bar{c}_{11}\alpha_i^2-g_1\right)\left(\bar{e}_{15}\alpha_i^2+g_6\right)-g_3g_6\alpha_i^2t_L^2}{t_L^2\left(g_2\left(\bar{e}_{15}\alpha_i^2+g_6\right)+g_3\left(g_5-\alpha_i^2\right)\right)}$$

$$e_i=\frac{\left(\bar{c}_{11}\alpha_i^2-g_1\right)\left(g_5-\alpha_i^2\right)+g_2^2\alpha_i^2t_L^2}{t_L^2\left(g_2\left(\bar{e}_{15}\alpha_i^2+g_6\right)+g_3\left(\alpha_i^2-g_5\right)\right)}. \tag{54}$$

Similarly solving the Eq. (49), we obtain the solution for the symmetric mode as

$$\bar{\psi}_n\left(r\right)=A_{4n}J_n\left(\alpha_4r\right)\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t}. \tag{55}$$

The boundary condition for a electro-elastic plate of polygonal cross-section is obtained as

(i) Stress free(unclamped edge)

$$\left(\sigma_{xx}\right)_i=\left(\sigma_{xy}\right)_i=\left(\sigma_{xz}\right)_i=\left(D_r\right)_i=0; \tag{56}$$

(ii) Rigidly fixed (clamped edge)

$$\left(u_r\right)_i=\left(u_\theta\right)_i=\left(E\right)_i=0, \tag{57}$$

where σ_{xx} is the normal stress, σ_{xy}, σ_{xz} are the shearing stresses, D_r is the electric potential as discussed in the section A. By using the same procedure as discussed in the section A, the boundary conditions (56) and (57) are transferred as

$$\sigma'_{xx}=c_{11}\cos^2\left(\theta-\gamma_i\right)+c_{12}\sin^2\left(\theta-\gamma_i\right)u_{,r}+r^{-1}\left(c_{11}\sin^2\left(\theta-\gamma_i\right)+c_{12}\cos^2\left(\theta-\gamma_i\right)\right)\left(u+v_{,\theta}\right)$$
$$+c_{66}\left(r^{-1}\left(v-u_{,\theta}\right)-v_{,r}\right)\sin 2\left(\theta-\gamma_i\right)+c_{13}w_{,z}+e_{31}E_{,zz}=0$$

$$\sigma'_{xy}=c_{66}\left(\left(u_{,r}-r^{-1}\left(v_{,\theta}+u\right)\right)\sin 2\left(\theta-\gamma_i\right)+\left(r^{-1}\left(u_{,\theta}-v\right)+v_{,r}\right)\cos 2\left(\theta-\gamma_i\right)\right)=0$$

$$\sigma'_{xz}=c_{44}\left(\left(u_{,z}+w_{,r}\right)\cos\left(\theta-\gamma_i\right)-\left(v_{,z}+r^{-1}w_{,\theta}\right)\sin\left(\theta-\gamma_i\right)\right)+e_{15}\left(E_{,r}\cos\left(\theta-\gamma_i\right)-\frac{1}{r}E_{,\theta}\sin\left(\theta-\gamma_i\right)\right)=0$$

$$D_x=0 \tag{58}$$

Substituting the Eq.v(53) in the Eq. (56) the boundary conditions are transformed for stress free polygonal cross-sectional plate is obtained as

$$\left[\left(S'_{xx}\right)_i+\left(\bar{S}'_{xx}\right)_i\right]\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t}$$

$$\left[\left(S'_{xy}\right)_i+\left(\bar{S}'_{xy}\right)_i\right]\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t}$$

$$\left[\left(S'_{xz}\right)_i+\left(\bar{S}'_{xz}\right)_i\right]\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t}$$

$$\left[\left(E'_x\right)_i+\left(\overline{E'}_x\right)_i\right]\sin\left(m\pi\zeta\right)\sin n\theta e^{i\omega t}, \tag{59}$$

where $S'_{xx}=0.5\left(A_{10}p_0^1+A_{20}p_0^2+A_{30}p_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}p_n^1+A_{2n}p_n^2+A_{3n}p_n^3+A_{4n}p_n^4\right)$

$$S'_{xy}=0.5\left(A_{10}q_0^1+A_{20}q_0^2+A_{30}q_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}q_n^1+A_{2n}q_n^2+A_{3n}q_n^3+A_{4n}q_n^4\right)$$

$$S'_{xz}=0.5\left(A_{10}r_0^1+A_{20}r_0^2+A_{30}r_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}r_n^1+A_{2n}r_n^2+A_{3n}r_n^3+A_{4n}r_n^4\right)$$

$$E'_x=0.5\left(A_{10}s_0^1+A_{20}s_0^2+A_{30}s_0^3\right)+\sum_{n=1}^{\infty}\left(A_{1n}s_n^1+A_{2n}s_n^2+A_{3n}s_n^3+A_{4n}s_n^4\right) \tag{60}$$

$$\overline{S'}_{xx}=0.5\overline{p}_0^4\overline{A}_{40}+\sum_{n=1}^{\infty}\left(\overline{A}_{1n}\overline{p}_n^1+\overline{A}_{2n}\overline{p}_n^2+\overline{A}_{3n}\overline{p}_n^3+\overline{A}_{4n}\overline{p}_n^4\right)$$

$$\overline{S'}_{xy}=0.5\overline{q}_0^4\overline{A}_{40}+\sum_{n=1}^{\infty}\left(\overline{A}_{1n}\overline{q}_n^1+\overline{A}_{2n}\overline{q}_n^2+\overline{A}_{3n}\overline{q}_n^3+\overline{A}_{4n}\overline{q}_n^4\right)$$

$$\overline{E'}_x=0.5\overline{s}_0^4\overline{A}_{40}+\sum_{n=1}^{\infty}\left(\overline{A}_{1n}\overline{s}_n^1+\overline{A}_{2n}\overline{s}_n^2+\overline{A}_{3n}\overline{s}_n^3+\overline{A}_{4n}\overline{s}_n^4\right) \tag{61}$$

The boundary conditions along the irregular shape cannot be satisfied directly. To satisfy the boundary conditions, the Fourier expansion collocation method is applied along the boundary. Performing the Fourier series expansion to the transformed expression in Eq. (56) along the boundary, the boundary conditions are expanded in the form of double Fourier series for symmetric and antisymmetric modes of vibrations. For the symmetric mode, the equation which satisfies the boundary condition, is obtained in matrix form as follows

$$\begin{bmatrix}
P_{00}^1 & P_{00}^2 & P_{00}^3 & P_{01}^1 & \cdots & P_{0N}^1 & P_{01}^2 & \cdots & P_{0N}^2 & P_{01}^3 & \cdots & P_{0N}^3 & P_{01}^4 & \cdots & P_{0N}^4\\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots\\
P_{N0}^1 & P_{N0}^2 & P_{N0}^3 & P_{N1}^1 & \cdots & P_{NN}^1 & P_{N1}^2 & \cdots & P_{NN}^2 & P_{N1}^3 & \cdots & P_{NN}^3 & P_{N1}^4 & \cdots & P_{NN}^4\\
Q_{00}^1 & Q_{00}^2 & Q_{00}^3 & Q_{01}^1 & \cdots & Q_{0N}^1 & Q_{01}^2 & \cdots & Q_{0N}^2 & Q_{01}^3 & \cdots & Q_{0N}^3 & Q_{01}^4 & \cdots & Q_{0N}^4\\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots\\
Q_{N0}^1 & Q_{N0}^2 & Q_{N0}^3 & Q_{N1}^1 & \cdots & Q_{NN}^1 & Q_{N1}^2 & \cdots & Q_{NN}^2 & Q_{N1}^3 & \cdots & Q_{NN}^3 & Q_{N1}^4 & \cdots & Q_{NN}^4\\
R_{00}^1 & R_{00}^2 & R_{00}^3 & R_{01}^1 & \cdots & R_{0N}^1 & R_{01}^2 & \cdots & R_{0N}^2 & R_{01}^3 & \cdots & R_{0N}^3 & R_{01}^4 & \cdots & R_{0N}^4\\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots\\
R_{N0}^1 & R_{N0}^2 & R_{N0}^3 & R_{N1}^1 & \cdots & R_{NN}^1 & R_{N1}^2 & \cdots & R_{NN}^2 & R_{N1}^3 & \cdots & R_{NN}^3 & R_{N1}^4 & \cdots & R_{NN}^4\\
S_{00}^1 & S_{00}^2 & S_{00}^3 & S_{01}^1 & \cdots & S_{0N}^1 & S_{01}^2 & \cdots & S_{0N}^2 & S_{01}^3 & \cdots & S_{0N}^3 & S_{01}^4 & \cdots & S_{0N}^4\\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots\\
S_{N0}^1 & S_{N0}^2 & S_{N0}^3 & S_{N1}^1 & \cdots & S_{NN}^1 & S_{N1}^2 & \cdots & S_{NN}^2 & S_{N1}^3 & \cdots & S_{NN}^3 & S_{N1}^4 & \cdots & S_{NN}^4
\end{bmatrix}
\begin{bmatrix}
\widehat{A_{10}}\\ \widehat{A_{20}}\\ \widehat{A_{30}}\\ \widehat{A_{11}}\\ \vdots\\ \widehat{A_{1N}}\\ \widehat{A_{21}}\\ \vdots\\ \widehat{A_{2N}}\\ \widehat{A_{31}}\\ \vdots\\ \widehat{A_{3N}}\\ \widehat{A_{41}}\\ \vdots\\ \widehat{A_{4N}}
\end{bmatrix}=0 \tag{62}$$

where

$$P_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}p_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$

$$Q_{mn}^j=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i}q_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$

$$R_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} r_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$

$$S_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} s_n^j\left(R_i,\theta\right)\cos m\theta d\theta \tag{63}$$

Similarly, for the antisymmetric mode, we get

$$\begin{bmatrix}
\overline{P}_{10}^{4} & \overline{P}_{11}^{1} & \cdots & \overline{P}_{1N}^{1} & \overline{P}_{11}^{2} & \cdots & \overline{P}_{1N}^{2} & \overline{P}_{11}^{3} & \cdots & \overline{P}_{1N}^{3} & \overline{P}_{11}^{4} & \cdots & \overline{P}_{1N}^{4} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{P}_{N0}^{4} & \overline{P}_{N1}^{1} & \cdots & \overline{P}_{NN}^{1} & \overline{P}_{N1}^{2} & \cdots & \overline{P}_{NN}^{2} & \overline{P}_{N1}^{3} & \cdots & \overline{P}_{NN}^{3} & \overline{P}_{N1}^{4} & \cdots & \overline{P}_{NN}^{4} \\
\overline{Q}_{10}^{4} & \overline{Q}_{11}^{1} & \cdots & \overline{Q}_{1N}^{1} & \overline{Q}_{11}^{2} & \cdots & \overline{Q}_{1N}^{2} & \overline{Q}_{11}^{3} & \cdots & \overline{Q}_{1N}^{3} & \overline{Q}_{11}^{4} & \cdots & \overline{Q}_{1N}^{4} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{Q}_{N0}^{4} & \overline{Q}_{N1}^{1} & \cdots & \overline{Q}_{NN}^{1} & \overline{Q}_{N1}^{2} & \cdots & \overline{Q}_{NN}^{2} & \overline{Q}_{N1}^{3} & \cdots & \overline{Q}_{NN}^{3} & \overline{Q}_{N1}^{4} & \cdots & \overline{Q}_{NN}^{4} \\
\overline{R}_{10}^{4} & \overline{R}_{11}^{1} & \cdots & \overline{R}_{1N}^{1} & \overline{R}_{11}^{2} & \cdots & \overline{R}_{1N}^{2} & \overline{R}_{11}^{3} & \cdots & \overline{R}_{1N}^{3} & \overline{R}_{11}^{4} & \cdots & \overline{R}_{1N}^{4} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{R}_{N0}^{4} & \overline{R}_{N1}^{1} & \cdots & \overline{R}_{NN}^{1} & \overline{R}_{N1}^{2} & \cdots & \overline{R}_{NN}^{2} & \overline{R}_{N1}^{3} & \cdots & \overline{R}_{NN}^{3} & \overline{R}_{N1}^{4} & \cdots & \overline{R}_{NN}^{4} \\
\overline{S}_{10}^{4} & \overline{S}_{11}^{1} & \cdots & \overline{S}_{1N}^{1} & \overline{S}_{11}^{2} & \cdots & \overline{S}_{1N}^{2} & \overline{S}_{11}^{3} & \cdots & \overline{S}_{1N}^{3} & \overline{S}_{11}^{4} & \cdots & \overline{S}_{1N}^{4} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{S}_{N0}^{4} & \overline{S}_{N1}^{1} & \cdots & \overline{S}_{NN}^{1} & \overline{S}_{N1}^{2} & \cdots & \overline{S}_{NN}^{2} & \overline{S}_{N1}^{3} & \cdots & \overline{S}_{NN}^{3} & \overline{S}_{N1}^{4} & \cdots & \overline{S}_{NN}^{4}
\end{bmatrix}
\begin{bmatrix}
\widehat{\overline{A}}_{40} \\ \widehat{\overline{A}}_{11} \\ \vdots \\ \widehat{\overline{A}}_{1N} \\ \widehat{\overline{A}}_{21} \\ \vdots \\ \widehat{\overline{A}}_{21} \\ \widehat{\overline{A}}_{31} \\ \vdots \\ \widehat{\overline{A}}_{3N} \\ \widehat{\overline{A}}_{41} \\ \vdots \\ \widehat{\overline{A}}_{4N}
\end{bmatrix}=0 \tag{64}$$

where $\overline{P}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{p}_n^j\left(R_i,\theta\right)\sin m\theta d\theta$

$$\overline{Q}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{q}_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$

$$\overline{R}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{r}_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$

$$\overline{S}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \overline{s}_n^j\left(R_i,\theta\right)\sin m\theta d\theta, \tag{65}$$

where j=1,2,3 and 4, I is the number of segments, R_i is the coordinate r at the boundary and N is the terms in the Fourier series. The frequency equation for determining the frequencies may be obtained by equating the coefficient of the system of Eq. (62) or Eq. (64) to zero.

6. Numerical results and discussion

The frequency equations obtained in symmetric and antisymmetric cases given in Eq. (46) and (47) are analyzed numerically for magneto electro elastic plate of polygonal (triangular, square, pentagonal and hexagonal) cross-sections. The material properties of the electro-magnetic material based on graphical results of Aboudi [33] are $c_{11}=218\times10^{9}\,N/m^2$, $c_{12}=120\times10^{9}\,N/m^2$, $c_{13}=120\times10^{9}\,N/m^2$, $c_{33}=215\times10^{9}\,N/m^2$, $c_{44}=50\times10^{9}\,N/m^2$, $c_{66}=49\times10^{9}\,N/m^2$, $e_{15}=0$, $e_{31}=-2.5\,C/m^2$, $e_{33}=7.5\,C/m^2$, $q_{15}=200\,C/m^2$, $q_{31}=265\,C/m^2$, $q_{33}=345\,C/m^2$, $\varepsilon_{11}=0.4\times10^{-9}\,C/Vm$, $\varepsilon_{33}=5.8\times10^{-9}\,C/Vm$,

$\mu_{11} = -200 \times 10^{-6}\, Ns^2/C^2$, $\mu_{33} = 95 \times 10^{-6}\, Ns^2/C^2$, $m_{11} = 0.0074 \times 10^{-9}\, Ns/VC$, $m_{33} = 2.82 \times 10^{-9}\, Ns/VC$.

The geometric relations for the polygonal cross-sections given by Nagaya [32] as

$$R_i/b = \left[\cos(\theta - \gamma_i)\right]^{-1}, \tag{66}$$

where b is the apothem. The relation given in Eq. (66) is used directly for the numerical calculation. The dimensionless wave numbers, which are complex in nature, are computed by fixing Ω for $0 < \Omega \leq 1.0$ using secant method (applicable for complex roots). The basic independent modes like longitudinal and flexural modes of vibration are analyzed and the corresponding non-dimensional wave numbers are computed. The polygonal cross-sectional bar in the range $\theta = 0$ and $\theta = \pi$ is divided into many segments for convergence of wave number in such a way that the distance between any two segments is negligible. The computation of Fourier coefficients given in Eq. (44) is carried out using the five point Gaussian quadrature.

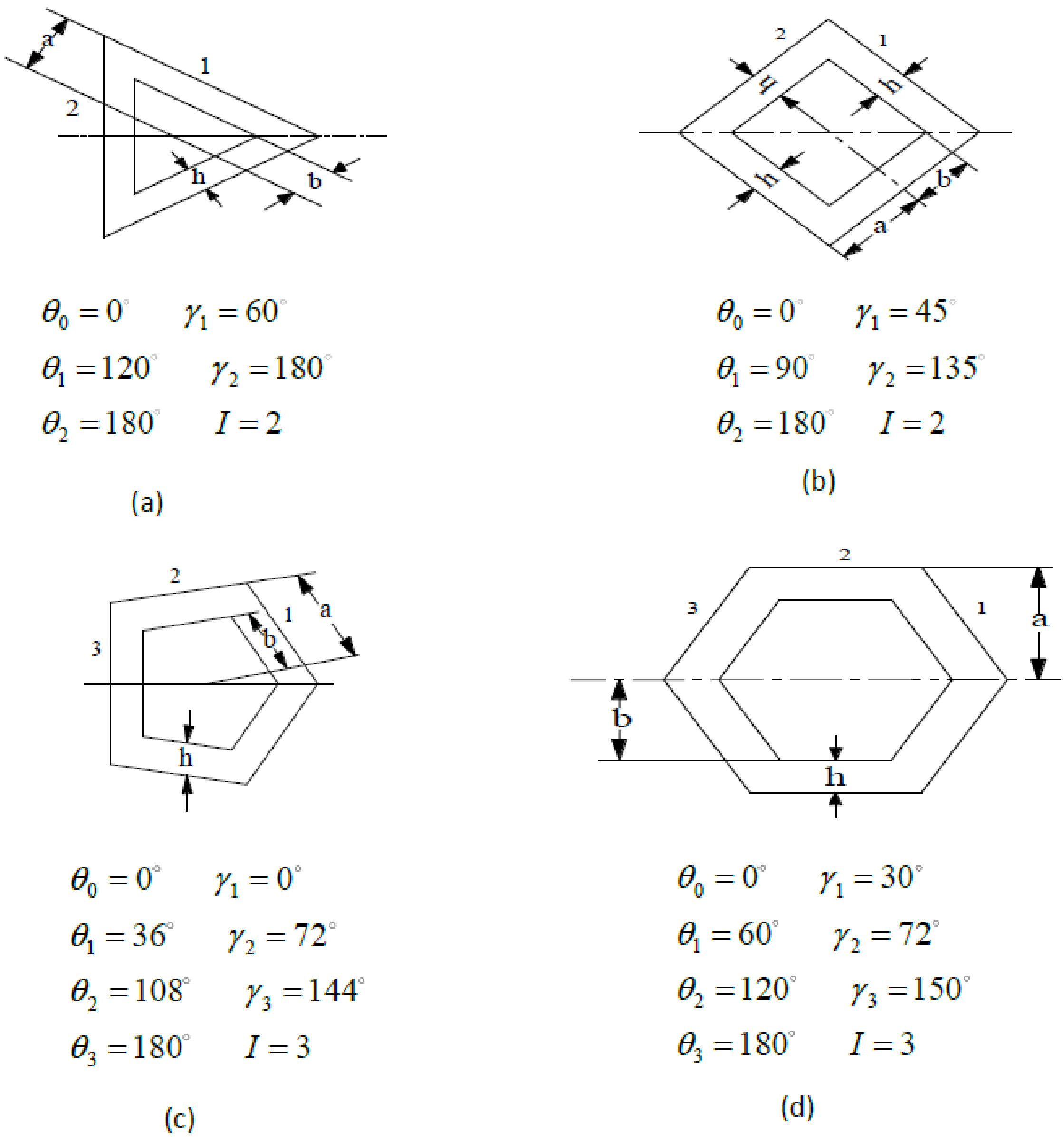

Fig. 1. Geometry of ring shaped polygonal plates

Longitudinal modes of polygonal plates. In case of longitudinal vibration of square and hexagonal cross-sectional plates, the displacements are symmetrical about both major and minor axes, since both the cross-sections are symmetric about both the axes. Therefore the frequency equation is obtained by choosing both terms of n and m as 0, 2, 4, 6… in Eq. (46). During flexural motion, the displacements are anti-symmetrical about the major axis and symmetrical about the minor axis. Hence the frequency equation is obtained by choosing n, m=1, 3, 5 in Eq. (46).

Flexural modes of polygonal plates. In flexural mode of square and hexagonal cross-section, the vibration and displacements are antisymmetrical about the major axis and symmetrical about the minor axis The vibrational displacements are symmetrical about the x axis for the longitudinal mode and anti-symmetrical about the y axis for the flexural mode in the triangular and pentagonal cross-sectional plates, since the cross-section is symmetric about only one axis. Therefore n and m are chosen as 0, 1, 2, 3… in Eq. (47) for the longitudinal mode and n, m=1, 2, 3 … in Eq. (47) for the flexural mode.

Dispersion analysis. The variation of circumferential stress $\sigma_{\theta\theta}$ with the non-homogeneous parameter m is discussed for different cross section of the magneto electro elastic plate in Fig. 2. It is clear that, the circumferential stress propagation behavior which is caused by the non-homogeneous parameter m is decreasing in all the cross section of the plates. Fig. 3 shows the variation of the radial stress σ_{rr} with respect to the non-homogeneous parameter m of the magneto electro elastic plate for various cross section of the magneto electro elastic plate. From the curves in Fig. 3, it is clear that the radial stresses are higher in lower non-homogeneous parameter m and decreases slowly in the remaining range with small oscillation in the hexagonal plate. The parameter m is effective in the stress distribution of the entire cross sectional plate.

Figure 4 depicts the variation of the radial strain e_{rr} with respect to the non-homogeneous parameter m of the magneto electro elastic polygonal cross sectional plate. In Fig. 4, the radial strain obtain the positive values in the range $0 \le m \le 0.075$ for all cross sectional plates, then the radial strain distribution goes on increasing and vanishes on the domain $m \ge 0.25$.The trend is same in circumferential strain $e_{\theta\theta}$ in Fig. 5 for all type of cross sectional plates, except there is a small deviation in the starting range of the non-homogeneous parameter m.

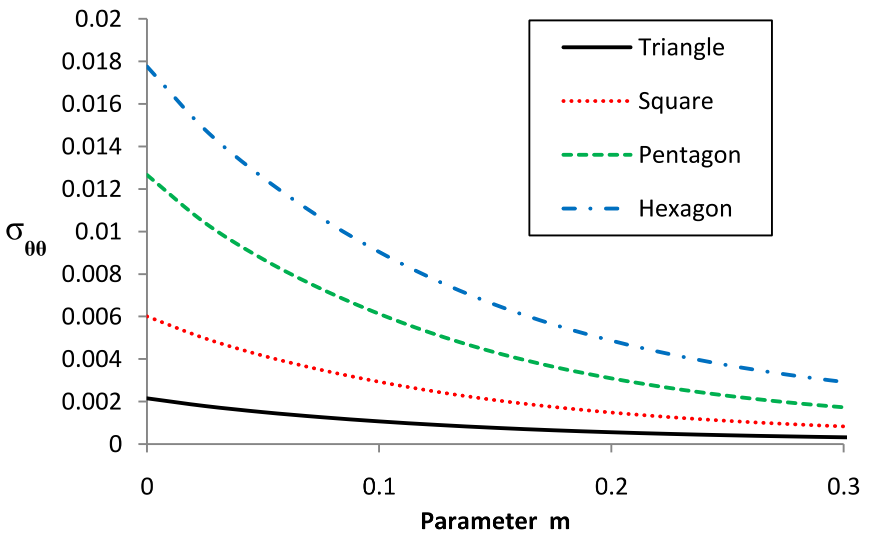

Fig. 2. Variation of circumferential stress versus parameter m for different cross sections of the plate

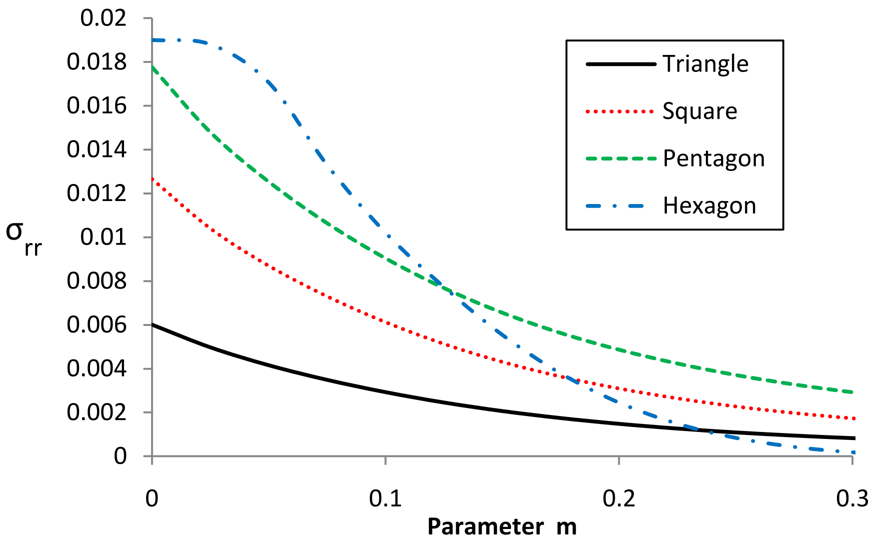

Fig. 3. Variation of radial stress versus parameter m for different cross sections of the plate

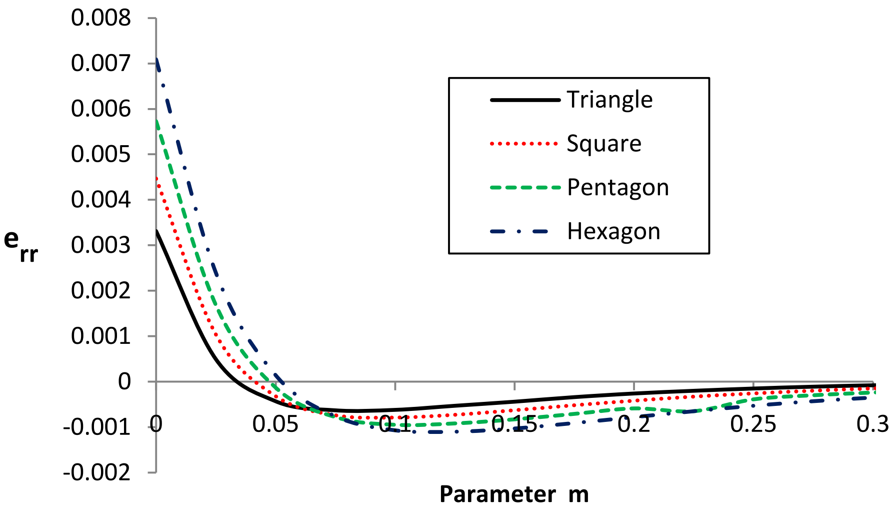

Fig. 4. Variation of radial strain versus parameter m for different cross sections of the plate

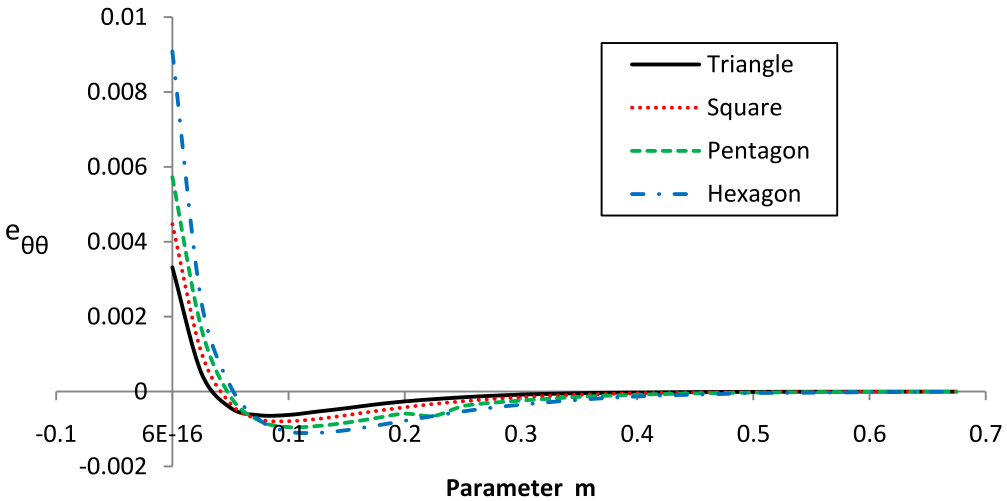

Fig. 5. Variation of circumferential strain versus parameter m for different cross sections of the plate

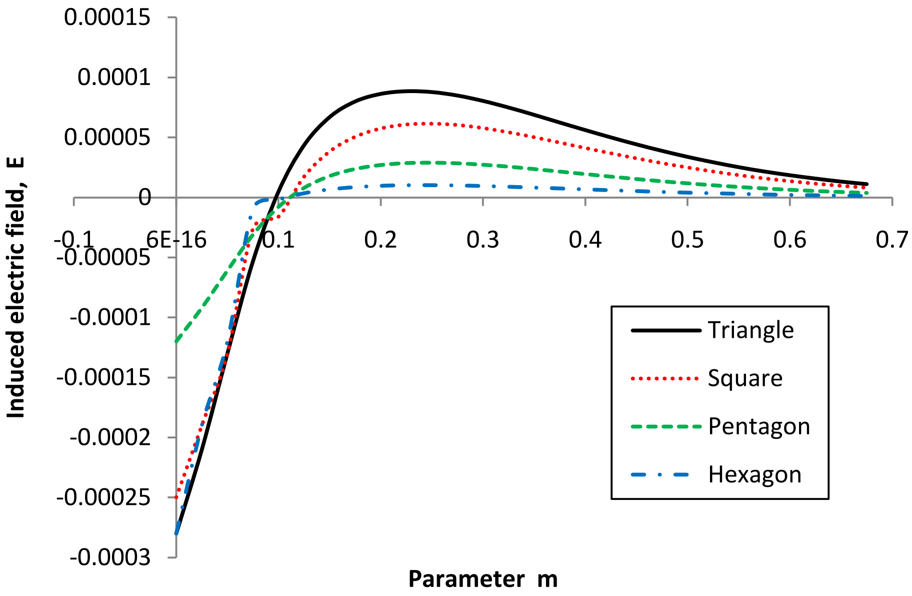

Fig. 6. Variation of induced electric field versus parameter m for different cross sections of the plate

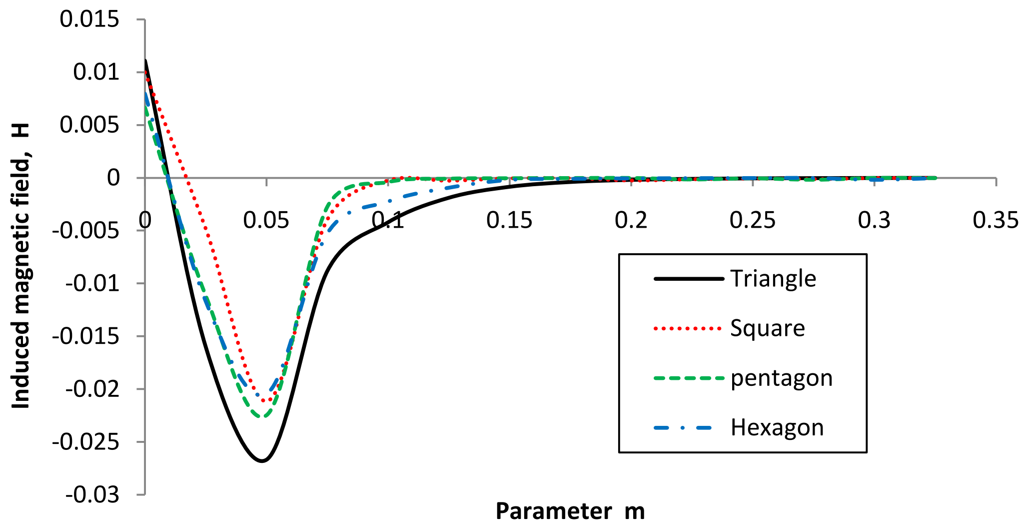

Fig. 7. Variation of induced magnetic field versus parameter m for different cross sections of the plate

A graph is drawn between the variations of induced electric field versus the non-homogeneous parameter m of magneto electro elastic plate of polygonal cross sections in Fig.6. From the Fig.6, it is clear that the displacement of induced electrical energy is getting negative values in the range $0 \leq m \leq 0.1$, but for the higher values of m it becomes constant for all the cross sections of the plate. The transfer of electrical energy is higher in the lower values of the parameter m as compared to the higher values and this cross over point represents the transfer of electrical energy between modes of vibration of polygonal plates. The variation of the induced magnetic field versus the non-homogeneous parameter m of magneto electro elastic polygonal plates is analyzed in Fig. 7. From these curves it is clear that in the entire cross sectional plates, the induced magnetic field takes negative values in the range $0.01 \leq m \leq 0.15$ but for $m \geq 0.15$ slowly it vanishes.

7. Conclusion

The effect of magnetic field and non-homogeneity in a piezoelectric plate of polygonal cross sections is studied using the linear theory of elasticity. The wave equation of motion based on two-dimensional theory of elasticity is applied under the plane strain assumption of plate of polygonal shape, composed of homogeneous transversely isotropic material. The frequency equations are obtained by satisfying the irregular boundary conditions of the polygonal plate using Fourier expansion collocation method. The analytical results obtained in the physical domain have been computed numerically for a magneto electro elastic material. The numerically analyzed results for the stress, strain, displacements and induced electric and magnetic fields have been presented graphically. The polygonal plates, as structural elements, are widely used in construction of oil pipes, submarine and flight structures to ensure the strength and reliability, acted upon by nonuniform loads.

Appendix A

$$e_n^1 = \begin{bmatrix} \left(\beta(\beta-1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\right)\left(\bar{L}+\sin^2(\theta-\gamma_i)\right)-\left(\beta(\beta+1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\right) \\ \left(\bar{L}+\cos^2(\theta-\gamma_i)\right)+(\alpha r)^2\left(\left(1+\bar{L}\right)\cos^2(\theta-\gamma_i)+\bar{L}\sin^2(\theta-\gamma_i)\right)J_\beta(\alpha r) \end{bmatrix}\cos n\theta$$

$$-n\left\{(\beta-1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\right\}\sin 2(\theta-\gamma_i)\sin n\theta$$

$$e_n^1 = \begin{bmatrix} \left(\beta(\beta-1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\right)\left(\bar{L}+\sin^2(\theta-\gamma_i)\right)-\left(\beta(\beta+1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\right) \\ \left(\bar{L}+\cos^2(\theta-\gamma_i)\right)+(\alpha r)^2\left(\left(1+\bar{L}\right)\cos^2(\theta-\gamma_i)+\bar{L}\sin^2(\theta-\gamma_i)\right)J_\beta(\alpha r) \end{bmatrix}\cos n\theta$$

$$-n\left\{(\beta-1)J_\beta(\alpha r)+(\alpha r)J_{\beta+1}(\alpha r)\right\}\sin 2(\theta-\gamma_i)\sin n\theta$$

$$e_n^2 = 0 \qquad e_n^3 = 0$$

$$e_n^4 = \begin{bmatrix} \left(n(\delta-1)J_\delta(kr)+(kr)J_{\delta+1}(kr)\right)\cos 2(\theta-\gamma_i)\cos n\theta \\ -\left(\delta\left(\frac{\delta+1}{2}\right)+\left(\frac{n^2-(kr)^2}{2}\right)J_\delta(\alpha r)+(\alpha r)J_{\delta+1}(\alpha r)\right) \end{bmatrix}\sin 2(\theta-\gamma_i)\sin n\theta$$

$$f_n^1 = \left[2\left(\beta J_\beta(\alpha r)-(\alpha r)J_{\beta+1}(\alpha r)\right)+\left((\alpha r)^2-\beta^2-n^2\right)J_\beta(\alpha r)\right]\cos n\theta\sin 2(\theta-\gamma_i)$$

$$+2n\left\{(\beta-1)J_\beta(\alpha r)-(\alpha r)J_{\beta+1}(\alpha r)\right\}\cos 2(\theta-\gamma_i)\sin n\theta$$

$$f_n^2 = 0 \qquad f_n^3 = 0$$

$$f_n^4 = \begin{bmatrix} 2n\left(\delta J_\delta(kr)-(kr)J_{\delta+1}(kr)\right)\cos n\theta\sin 2(\theta-\gamma_i) \\ +2\left(\delta J_\delta(kr)-(kr)J_{\delta+1}(kr)\right)+\left((kr)^2-\delta^2-n^2\right)J_\delta(\alpha r) \end{bmatrix}\sin n\theta\cos 2(\theta-\gamma_i)$$

$$+2n\left\{(\beta-1)J_\beta(\alpha r)-(\alpha r)J_{\beta+1}(\alpha r)\right\}\cos 2(\theta-\gamma_i)\sin n\theta$$

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