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THERMOELECTRIC VISCOELASTIC SPHERICAL CAVITY WITH MEMORY-DEPENDENT DERIVATIVE

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Abstract. We apply the memory-dependent derivative theory of thermoelasticity to the one-dimensional problem for a viscoelastic spherical cavity subjected to thermal loading. The predictions of the theory are discussed and compared with those for the coupled theory of thermoelasticity.

Keywords: thermo-viscoelasticity, thermoelectric properties, memory-dependent derivative, spherical cavity, Laplace transforms, numerical results

1. Introduction

Biot [1] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on temperature. Lord and Shulman [2] introduced the theory of generalized thermoelasticity in one relaxation time by using the Maxwell-Cattaneo law of heat conduction instead of the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity.

The uniqueness of solution for thermoelasticity theory was proved under different conditions by Ignaczak [3], Sherief [4], Chandrasekharaiah [5], and thermo-viscoelasticity theory by Ezzat and El-Karamany [6,7]. Ezzat and Awad [8,9] extended this theory to deal with micropolar materials.

An immediate change amongst electricity and heat by utilizing thermoelectric materials has pulled in much consideration as a result of their potential applications in Peltier coolers also, thermoelectric power generators [10]. Among the commitments in continuum mechanics of thermoelectric materials are crafted by Shercliff [11] and Ezzat and Youssef [12,13] in MHD.

Differential equations of fractional order have been the focal point of numerous examinations because of their continuous appearance in different applications in liquid mechanics, viscoelasticity, science, material science, and building. Povstenko [14] investigated new thermoelasticity models that use fractional derivatives. The fractional-order theory of thermoelasticity was derived by Sherief et al. [15]. As of late, Ezzat [16-19] and Ezzat, El-Karamany [20,21] and Ezzat et al. [22,23] set up another model of partial heat conduction equation utilizing the Taylor-Riemann series extension of time-fractional order.

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Wang and Li [24] proposed the memory-dependent derivative (MDD) to characterize the memory effect of systems and materials. Yu et al. [25] introduced the first-order MDD model to describe the rate of heat flux in the Lord and Shulman generalized thermoelasticity. In this model, the heat transfer equation has been modified, which may be better than fractional equations. It may be mentioned that the definition of MDD is more intuitive in the perception of physical importance, and therefore, the resultant differential equations based on memory-dependent are more efficient in real-world applications [25]. Recently, memory-dependent heat transfer has been applied to solve many related thermoelastic and thermo-viscoelastic problems [26–28].

Ezzat et al. [29] proposed the generalized Ohm and Fourier laws for elasto-thermoelectric materials subjected to MDD heat transfer when the medium is permeated by an external magnetic field, as

$$\boldsymbol{J} = \sigma_o \left(\boldsymbol{E} + \frac{\partial \boldsymbol{u}}{\partial t} \wedge \mu_o \boldsymbol{H} - k_o \nabla T \right), \tag{1}$$

$$\boldsymbol{q} + \omega D_\omega \boldsymbol{q} = - k \nabla T + \pi_o \boldsymbol{J}, \tag{2}$$

where $\boldsymbol{q}$ is the heat flux vector and ω is the time delay.

Among the few works devoted to MDD applications of heat transfer equation for thermoelectric materials, we can refer to the survey of Ezzat and El-Bary [30], Ezzat et al. [31], Hendy et al. [32] and Ezzat [33].

In light of the advantage in describing the memory effect of thermoelectric materials, we solve a 1D thermal shock problem for a viscoelastic spherical cavity by using the memory-dependent derivative theory. The solution is obtained for different values of the parameters of the MDD model. Some comparisons are made and shown in figures to estimate the effects of the time-delay parameter for different forms of kernel function on all studied fields.

2. Physical problem

Let (r, ψ, φ) denote spherical polar coordinates. We shall consider a homogeneous, isotropic, thermoelectric viscoelastic medium occupying the region $a \le r < \infty$, where a is the radius of the spherical cavity. The surface of the cavity is taken to be traction-free and subjected to a thermal shock that is a function of time. A constant magnetic field of intensity $\boldsymbol{H}$ with components $(0, H_o, 0)$ permeates the medium in the absence of an external electric field $\boldsymbol{E}$. Since no external electric field is applied, and the effect of polarization of the ionized medium can be neglected, it follows that the total electric field $\boldsymbol{E}$ vanishes identically inside the medium [34].

Because of spherical symmetry, the displacement vector $\boldsymbol{u}$ will have the components

$$u_r = u(r,t), \quad u_\psi = 0, \quad u_\varphi = 0. \tag{3}$$

The components of the electric current density vector $\boldsymbol{J}$ are [17]

$$J_r = - \sigma_o k_o \frac{\partial T}{\partial r}, \quad u_\psi = 0, \quad u_\varphi = \mu_o H_o \frac{\partial u}{\partial t}. \tag{4}$$

The components of the electromagnetic induction vector are given by [18]

$$B_\psi = \mu_o H_o = B_o = \text{constant}, \quad B_r = B_\varphi = 0. \tag{5}$$

The Lorentz force components are given by [19]

$$F_r = - \sigma_o \mu_o^2 H_o^2 \frac{\partial u}{\partial t}, \quad F_\psi = 0, \quad F_\varphi = 0. \tag{6}$$

The components of strain tensor are given by [37]:

$$e_{rr} = \frac{\partial u}{\partial r}, \; e_{\psi\psi} = e_{\varphi\varphi} = \frac{u}{r}, \; e_{r\psi} = e_{r\varphi} = e_{\psi\varphi} = 0. \tag{7}$$

The figure-of-merit ZT_o at some reference temperature T_o [35]:

$$ZT_o = \frac{\sigma_o k_o^2}{k} T_o. \tag{8}$$

The first Thomson relation at T_o [36]:

$$\pi_o = k_o \, T_o, \tag{9}$$

where π_o is the Peltier coefficient at T_o.

The components of the stress tensor are given by [38, 39]

$$\sigma_{rr} = K_o e + \left(\frac{2}{3}\widehat{R} + K_o\right)\left(\frac{\partial u}{\partial r}\right) - \gamma (T - T_o), \tag{10}$$

$$\sigma_{\psi\psi} = \sigma_{\varphi\varphi} = K_o e + \left(\frac{2}{3}\widehat{R} + K_o\right)\left(\frac{u}{r}\right) - \gamma (T - T_o), \tag{11}$$

$$\sigma_{r\psi} = \sigma_{r\varphi} = \sigma_{\psi\varphi} = 0, \tag{12}$$

where e is the cubical dilatation given by

$$e = e_{rr} + e_{\psi\psi} + e_{\varphi\varphi} = \frac{\partial u}{\partial r} + \frac{2u}{r} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u), \tag{13}$$

and $\widehat{R}(t)$ is relaxation function given by

$$\widehat{R}(t) = 2\mu\left[1 - A^* \int_0^t e^{-\beta^* t} t^{\alpha^* - 1} dt\right], \tag{14}$$

where α^*, β^*, and A^* are non-dimensional empirical constants and $\Gamma(\alpha^*)$ is the Gamma function, $0 < \alpha^* < 1$, $\beta^* > 0$, $0 \le A^* < \dfrac{\beta^*}{\Gamma(\alpha^*)}$, $\widehat{R}(t) > 0$, $\dfrac{d}{dt}\widehat{R}(t) < 0$ [40].

The equation of motion is given by [41]:

$$\rho \frac{\partial^2 u}{\partial t^2} = \left(\frac{2}{3}\widehat{R} + K_o\right)\frac{\partial e}{\partial r} - \sigma_o B_o^2 \frac{\partial u}{\partial t} - \gamma \frac{\partial T}{\partial r}. \tag{15}$$

The MDD energy equation [28]:

$$k\left(1 + ZT_o\right)\nabla^2 T = \left(1 + \omega D_\omega\right)\left(\rho C_E \frac{\partial T}{\partial t} + \gamma T_o \frac{\partial e}{\partial t}\right), \tag{16}$$

where ∇^2 is the one-dimensional Laplace's operator in spherical polar coordinates, given by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

and in the memory-dependent derivative theory, the first order of function f which is essentially characterized in a vital type of a typical subordinate with a part work on a slipping interim [24]:

$$D_\omega f(t) = \frac{1}{\omega}\int_{t-\omega}^{t} K(t-\xi)\, f'(\xi)\, \mathrm{d}\xi,$$

where ω is the time delay and $K(t-\omega)$ is the kernel function in which they can be chosen freely.

Let us introduce the following non-dimensional variables:

$$r' = c_o \zeta_o r, \quad u' = c_o \zeta_o u, \quad t' = c_o{}^2 \zeta_o t, \quad \sigma' = \frac{\sigma}{K_o}, \quad \theta^* = \frac{\gamma(T - T_o)}{K_o}, \; q^* = \frac{\gamma}{k \rho c_o^3 \zeta_o} q \,,$$

$$\zeta_o = \rho C_E / k, \quad c_o^2 = \frac{K_o}{\rho}, \; \varepsilon = \frac{\gamma}{\rho C_E}, \quad M = \frac{\sigma_o \mu_o^2 H_o^2}{K_o \, \eta_o}, \; T_o = \frac{K}{\gamma}, \; \widehat{R}' = \frac{2}{3K_o} \widehat{R}.$$

The Equations (15), (16), (10) and (11) in non-dimensional form become

$$\frac{\partial^2 u}{\partial t^2} = \left(1 + \widehat{R}\right) \frac{\partial e}{\partial r} - M \frac{\partial u}{\partial t} - \frac{\partial \theta}{\partial r}, \tag{17}$$

$$(1 + Z T_o) \nabla^2 \theta = \left(1 + \omega D_\omega\right) \left(\frac{\partial \theta}{\partial t} + \varepsilon \frac{\partial e}{\partial t} \right), \tag{18}$$

$$\sigma_{rr} = e + \left(\widehat{R} + 1\right) \left(\frac{\partial u}{\partial r} \right) - \theta, \tag{19}$$

$$\sigma_{\psi\psi} = \sigma_{\varphi\varphi} = e + \left(\widehat{R} + 1\right) \left(\frac{u}{r} \right) - \theta. \tag{20}$$

The boundary conditions are taken as follows:

(i) The warm limit condition is that the outside of the cavity exposed to a warm stun that is a component of time

$$\theta(r,t) = f\,(t), \quad r = a. \tag{21}$$

(ii) The outside of the cavity are without footing (zero stress) i.e.

$$\sigma_{rr}(r,t) = 0, \quad r = a. \tag{22}$$

Condition (21) means that the surface of the cavity is traction-free, that is, there are no mechanical loads on the surface while Condition (22) means that the surface of the cavity is kept at a known temperature that is a function of time.

The initial conditions are taken to be homogeneous, that is, we take

$$u(r,t)\big|_{t=0} = 0, \quad \left.\frac{\partial u(r,t)}{\partial t}\right|_{t=0} = 0, \tag{23a}$$

$$\theta(r,t)\big|_{t=0} = 0, \quad \left.\frac{\partial \theta(r,t)}{\partial t}\right|_{t=0} = 0, \tag{23b}$$

$$\sigma_{rr}(r,t)\big|_{t=0} = 0, \quad \left.\frac{\partial \sigma_{rr}(r,t)}{\partial t}\right|_{t=0} = 0, \tag{23c}$$

$$\sigma_{\psi\psi}(r,t)\big|_{t=0} = 0, \quad \left.\frac{\partial \sigma_{\psi\psi}(r,t)}{\partial t}\right|_{t=0} = 0, \tag{23d}$$

$$\sigma_{\varphi\varphi}(r,t)\big|_{t=0} = 0, \quad \left.\frac{\partial \sigma_{\varphi\varphi}(r,t)}{\partial t}\right|_{t=0} = 0. \tag{23e}$$

3. The analytical solutions in the Laplace-transform domain

Playing out the Laplace transform characterized by the connection

$$\bar{g}(x,s) = L\{g(x,t)\} = \int_0^\infty \mathrm{e}^{-st} g(x,t)\, \mathrm{d}t$$

of both sides Eqs. (17)-(22), and using the initial Conditions (23), we obtain

$$s(s + M)\bar{u} = \beta^2 \frac{\partial \bar{e}}{\partial r} - \frac{\partial \bar{\theta}}{\partial r}, \tag{24}$$

$$\left(\nabla^2 - \varpi\right)\bar{\theta} = \varpi\,\varepsilon\bar{e}\ , \tag{25}$$

$$\bar{\sigma}_{rr} = \bar{e} + \beta^2\left(\frac{\partial \bar{u}}{\partial r}\right) - \bar{\theta}\ , \tag{26}$$

$$\bar{\sigma}_{\psi\psi} = \bar{\sigma}_{\varphi\varphi} = \bar{e} + \beta^2\left(\frac{\bar{u}}{r}\right) - \bar{\theta}\ , \tag{27}$$

$$\bar{R}(s) = \frac{4\mu}{3sK_o}\left[1 - \frac{A^*\Gamma(\alpha^*)}{(s+\beta^*)^{\alpha^*}}\right], \tag{28}$$

where

$$G(s) = (1-e^{-s\omega})(1-\frac{2n}{\omega s}+\frac{2m^2}{\omega^2 s^2}) - (m^2 - 2n + \frac{2m^2}{\omega s})\,e^{-s\omega}\ , \tag{29}$$

and $L\left\{\hat{R}\,\dfrac{\partial^2 u}{\partial x^2}\right\} = s\bar{R}(s)\dfrac{\partial^2 \bar{u}}{\partial x^2}$, $\beta^2 = 1 + s\bar{R}$, $\varpi = s\left(\dfrac{1+G(s)}{1+ZT_o}\right)$.

Applying the divergence operator on both sides of Equation (24) we obtain the equation of motion in the form

$$\left[\beta^2\,\nabla^2 - \ s(s+M\ )\right]\bar{e} = \nabla^2\bar{\theta}\ . \tag{30}$$

Removing $\bar{\theta}$ between Eqs. (25) and (30), we are getting

$$\left\{\beta^2\nabla^4 - \left[s(s+M\ ) + \varpi(\beta^2+\varepsilon)\right]\nabla^2 + \varpi\,s(s+M\ )\right\}\bar{e} = 0\ . \tag{31}$$

The above equation can be made as a factor

$$\left(\nabla^2 - \xi_1^2\right)\left(\nabla^2 - \xi_2^2\right)\bar{e} = 0\ , \tag{32}$$

where ξ_1, ξ_2 are the roots with positive real parts of the characteristic equation

$$\beta^2\xi^4 - \left[s(s+M\ ) + \varpi(\beta^2+\varepsilon)\right]\xi^2 + \varpi s(s+M\ ) = 0\ . \tag{33}$$

The roots of the characteristic equation are given by

$$\xi_{1,2}^2 = \frac{1}{2\beta^2}\left[s(s+M\ ) + \varpi(\beta^2+\varepsilon) \pm \sqrt{\left[\varpi(\beta^2+\varepsilon) - s(s+M\ )\right]^2 + 4s(s+M\ )\varpi\varepsilon}\,\right].$$

Because of linearity, Eq. (32) solution can be written as

$$\bar{e} = \bar{e}_1 + \bar{e}_2\ , \tag{34}$$

where

$$\left(\nabla^2 - \xi_1^2\right)\bar{e}_1 = 0, \quad \left(\nabla^2 - \xi_2^2\right)\bar{e}_2 = 0\ . \tag{35}$$

We consider the equation

$$\left(\nabla^2 - \xi^2\right)f\ = 0\ ,$$

this can be written as

$$\frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} - \xi^2 f\ = 0.$$

Taking the replacement

$$f\ = \frac{h}{\sqrt{r}}\ ,$$

the equation above is reduced to

$$r^2\frac{\mathrm{d}^2 h}{\mathrm{d}r^2} + r\,\frac{\mathrm{d}h}{\mathrm{d}r} - \left(r^2\xi^2 + \frac{1}{4}\right)h = 0.$$

The solution of this equation bounded at infinity has the form

$h = K_{1/2}(\xi r)$,

where $K_{1/2}(\cdot)$ is the modified Bessel function of the second kind of the order of 1/2.

The solution of Eq. (32) can be written on the basis of the above outcomes as

$$\bar{e} = \frac{1}{\sqrt{r}}\left[A\xi_1^2 K_{1/2}(\xi_1 r) + B\xi_2^2 K_{1/2}(\xi_2 r)\right], \tag{36}$$

where A and B are parameters depending on s.

Eliminating $\bar{e}$ between Equations (25) and (30), we obtain

$$\left(\nabla^2 - \xi_1^2\right)\left(\nabla^2 - \xi_2^2\right)\bar{\theta} = 0\,. \tag{37}$$

The solution of Eq. (32) that is compatible with Eqs. (25) and (36) is given by

$$\bar{\theta} = \frac{1}{\sqrt{r}}\left[A\left(\beta^2\,\xi_1^2 - s\,(s+M)\right)K_{1/2}(\xi_1 r) + B\left(\beta^2\,\xi_2^2 - s\,(s+M)\right)K_{1/2}(\xi_2 r)\right]. \tag{38}$$

Differentiating Eqs. (36) and (38) with respect to r and substituting the results into Eq. (24) gives

$$\bar{u} = -\frac{1}{\sqrt{r}}\left[A\xi_1 K_{3/2}(\xi_1 r) + B\xi_2 K_{3/2}(\xi_2 r)\right]. \tag{39}$$

Substituting from Equations (36), (38) and (39) into Equations (26), (27), we obtain

$$\begin{aligned}\bar{\sigma}_{rr} = \frac{1}{\sqrt{r}}\Bigg[&A\left([\xi_1^2 + s(s+M)]K_{1/2}(\xi_1 r) + \frac{2\beta^2}{r}\xi_1 K_{3/2}(\xi_1 r)\right)\\ &+ B\left([\xi_2^2 + s(s+M)]K_{1/2}(\xi_2 r) + \frac{2\beta^2}{r}\xi_2 K_{3/2}(\xi_2 r)\right)\Bigg],\end{aligned} \tag{40}$$

$$\begin{aligned}\bar{\sigma}_{\psi\psi} = \bar{\sigma}_{\varphi\varphi} = \frac{1}{\sqrt{r}}\Bigg[&A\left([(1-\beta^2)\xi_1^2 + s(s+M)]K_{1/2}(\xi_1 r) - \frac{\beta^2}{r}\xi_1 K_{3/2}(\xi_1 r)\right)\\ &+ B\left([(1-\beta^2)\xi_2^2 + s(s+M)]K_{1/2}(\xi_2 r) - \frac{\beta^2}{r}\xi_2 K_{3/2}(\xi_2 r)\right)\Bigg].\end{aligned} \tag{41}$$

The boundary conditions (21) and (22) can be written in the Laplace transform domain as

$$\bar{\theta}(r,s) = \bar{f}(s), \quad r = a\,, \tag{42}$$

$$\bar{\sigma}_{rr}(r,s) = 0, \quad r = a\,. \tag{43}$$

Using the boundary conditions (42) and (43) into Eqs. (38) and (40), we get

$$A\left(\beta^2\,\xi_1^2 - s\,(s+M)\right)K_{1/2}(\xi_1 a) + B\left(\beta^2\,\xi_2^2 - s\,(s+M)\right)K_{1/2}(\xi_2 a) = \sqrt{a}\,\bar{f}(s)\,, \tag{44}$$

$$\begin{aligned}&A\left([(1-\beta^2)\xi_1^2 + s(s+M)]K_{1/2}(\xi_1 a) - \frac{\beta^2}{a}\xi_1 K_{3/2}(\xi_1 a)\right)\\ &\quad + B\left([(1-\beta^2)\xi_2^2 + s(s+M)]K_{1/2}(\xi_2 a) - \frac{\beta^2}{a}\xi_2 K_{3/2}(\xi_2 a)\right) = 0.\end{aligned} \tag{45}$$

From now on, we shall utilize the exponential type of the modified Bessel functions of the second kind, in particular

$$K_{1/2}(z) = \sqrt{\frac{\pi}{2z}}e^{-z}, \quad K_{3/2}(z) = \sqrt{\frac{\pi}{2z}}\left(1 + \frac{1}{z}\right)e^{-z}. \tag{46}$$

By solving the system of two Equations (43) and (45) and using Eq. (46), we get the values of the two parameters A and B as

$$A=\frac{a}{\gamma_1}\sqrt{\frac{2}{\pi}}\left(\sqrt{\xi_1}\left[a^2[(1-\beta^2)\xi_1^2+s(s+M)]\;+\;\beta^2(a\xi_2+1)\right]e^{\xi_1 a}\right)\bar{f}(s)\,, \tag{47a}$$

$$B=-\frac{a}{\delta}\sqrt{\frac{2}{\pi}}\left(\sqrt{\xi_2}\left[a^2[(1-\beta^2)\xi_2^2+s(s+M)]+\beta^2(a\xi_2+1)\right]e^{\xi_2 a}\right)\bar{f}(s)\,, \tag{47b}$$

where $\delta=(\xi_1-\xi_2)\left(a^2[(1-\beta^2)\xi_1^2-s(s+M)]+\beta^2[\xi_1+\xi_2+as(s+M)+a\xi_1\xi_2]\right)$.

This completes the solution in the Laplace transform domain.

4. Inversion of Laplace transforms

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{f}(s)$ be the Laplace transform of a function $f(t)$. The inversion formula for Laplace transforms can be written as described in Honig and Hirdes [42]:

$$f(t)=\frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty}e^{ity}\,\bar{f}(d+iy)\,\mathrm{d}y\,,$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(s)$.

Expanding the function $h(t)=\exp(-dt)f(t)$ in a Fourier series in the interval [0, $2L$], we obtain the approximate formula [42]

$$f(t)\approx f_N(t)=\frac{1}{2}c_0+\sum_{k=1}^{N}c_k\,,\ \text{for}\ 0\le t\le 2\ell\,, \tag{48}$$

where

$$c_k=\frac{e^{dt}}{\ell}\mathrm{Re}\left[e^{ik\pi t/\ell}\,\bar{f}\left(d+ik\pi/\ell\right)\right]. \tag{49}$$

Two methods are used to reduce the total error. First, the "Korrektur" method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The "Korrektur" method uses the following formula to evaluate the function $f(t)$

$$f(t)=f_{NK}(t)=f_N(t)-e^{-2d\ell}f_{N'}(2\ell+t)\,. \tag{50}$$

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in (48). Let N be an odd natural number and let $s_m=\sum_{k=1}^{m}c_k$ be the sequence of partial sums of (48). We define the ε-sequence by

$\varepsilon_{0,m}=0,\ \varepsilon_{1,m}=s_m,\quad m=1,2,3,...$

and $\varepsilon_{n+1,m}=\varepsilon_{n-1,m+1}+1/\left(\varepsilon_{n,m+1}-\varepsilon_{n,m}\right),\quad n,m=1,2,3,....$

It can be shown that the sequence $\varepsilon_{1,1},\varepsilon_{3,1},...,\varepsilon_{N,1},...$ converges to $f(t)-c_0/2$ faster than the sequence of partial sums.

5. Numerical results and discussion

In this area, we intend to outline numerical consequences of the scientific articulations got in the past segment and explain the impact of time-delay ω and figure-of-merit ZT on the conduct of the field amounts for different forms of the kernel function. So as to translate the numerical calculations, we consider the material properties of Polymethyl Methacrylate (Plexiglas) which has wide applications in industry and prescription. Following the values of physical constants are shown in Table 1 [26]:

Table 1. Value of the constants

$\rho = 1.2\times10^3 kg/m^3$	$k = 0.55\ J/m.sec.K$	$T_o = 293$ K
$C_E = 1.4x10^3\ J/kg.K$	$\lambda = 453.7\times10^7 N/m^2$	$\mu = 194\times10^7 N/m^2$
$\gamma = 210\times10^4 N/m^2K$	$\eta_0 = 3.36\times10^6 sec/m^2$	$c_o = 2200\ m/sec$
$\alpha_T = 13\times10^{-5}$	$4\mu/3K_o = 0.8$	$\varepsilon = 0.12$
$\beta^* = 0.005$	$A^* = 0.106$	$\alpha^* = 0.5$
$\mu_o H_o = 1 Tesla$	$a = 1$	

The calculations were carried out for the function $f(t)$, which represents a time-dependent thermal shock:

$f(t) = t\,H(t)$ or $\bar{f}(s) = \dfrac{1}{s^2}$.

Thinking about the above physical information, we have assessed the numerical estimations of the field amounts with the assistance of a PC program created by utilizing the Fortran 90 programming language on a personal computer with a 17 processor. The amount of calculation (and hence the execution time) depends on several parameters within the program. First, there is a parameter "nsig" which is the number of significant digits defining the relative error as $(10)^{-\text{nsig}}$. We usually take nsig = 5. Near points of discontinuity of the function, the program might fail to converge, and we have to decrease nsig. Another parameter is the maximum number of terms in the Fourier series to be added within one saw-tooth of the ε-algorithm. This is taken as 10000. The last parameter is the number of saw-teeth of the ε-algorithm to be considered. This is taken as 50. All in all, the program evaluates the value of θ at 50 points in less than 2 minutes [43].

The calculations were completed for certain parameters, where an estimation of time, namely, t = 0. The investigation of the effects of the time delay and the figure-of-merit for different forms of kernel function as well as the magnetic number on thermoelectric material within the sight of a consistently attractive field was done in the former areas. Run-of-the-mill numerical outcomes have appeared in Figs. 1-8.

Figure 1 speaks to the dimensionless estimation of heat flux for a wide scope of outspread separation r $(0 \le r \le 1)$ and for different forms of the kernel function. In these figures, strong lines speak to the arrangement got in the casing of Biot theory $(\omega = 0)$ and broken lines represent the solution corresponding to using generalized electro-thermoelasticity $(\omega > 0)$ with MDD when the kernel function is taken as the form $[1-(t-\xi)/\omega]^2$, while dotted lines when the kernel function is $1-(t-\xi)$. We noticed that the maximum value of heat fluxes when $K = [1-(t-\xi)/\omega]^2$ and $K = 1-(t-\xi)$ are 2.02 and 1.8 and cut r-axis at $r = 1.0$. So, we learned from these figures that vital wonder saw in these assumes that the arrangement of any of the considered capacity in the new model is confined in a limited locale. Past this area, the varieties of these appropriations try not to occur. This implies to the arrangements concurring the new generalized hypothesis show the conduct of limited rates of wave spread.

Figure 2 indicates the variation of heat flux against the figure-of-merit for Biot theory $(\omega = 0)$ and for the generalized electro-thermoelasticity theory with MDD $(\omega > 0)$ when the kernel function has two forms, namely, $[1-(t-\xi)/\omega]^2$ and $1-(t-\xi)$. We noticed from this figure that the efficiency of a thermoelectric material figure-of-merit is proportional to the temperature of the solid particles and the choice of the kernel function forms has a significant effect on the heat flux field.

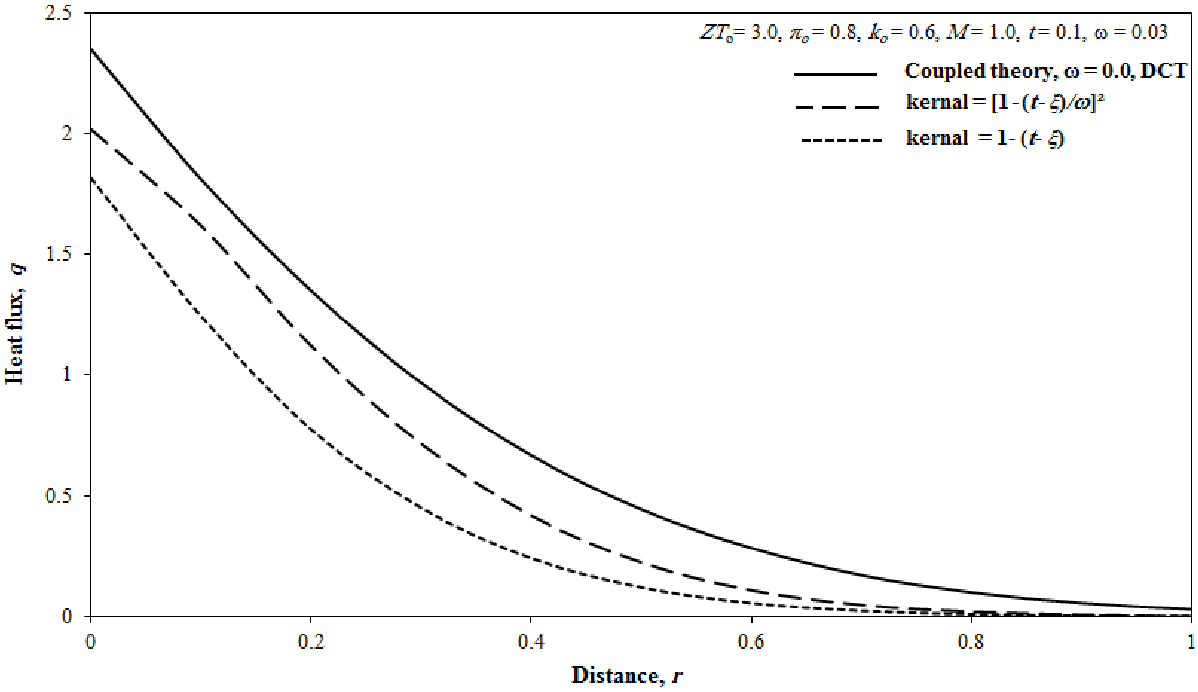

Fig. 1. The variation of heat flux vs. distance for different forms of kernal function $K(t, \xi)$

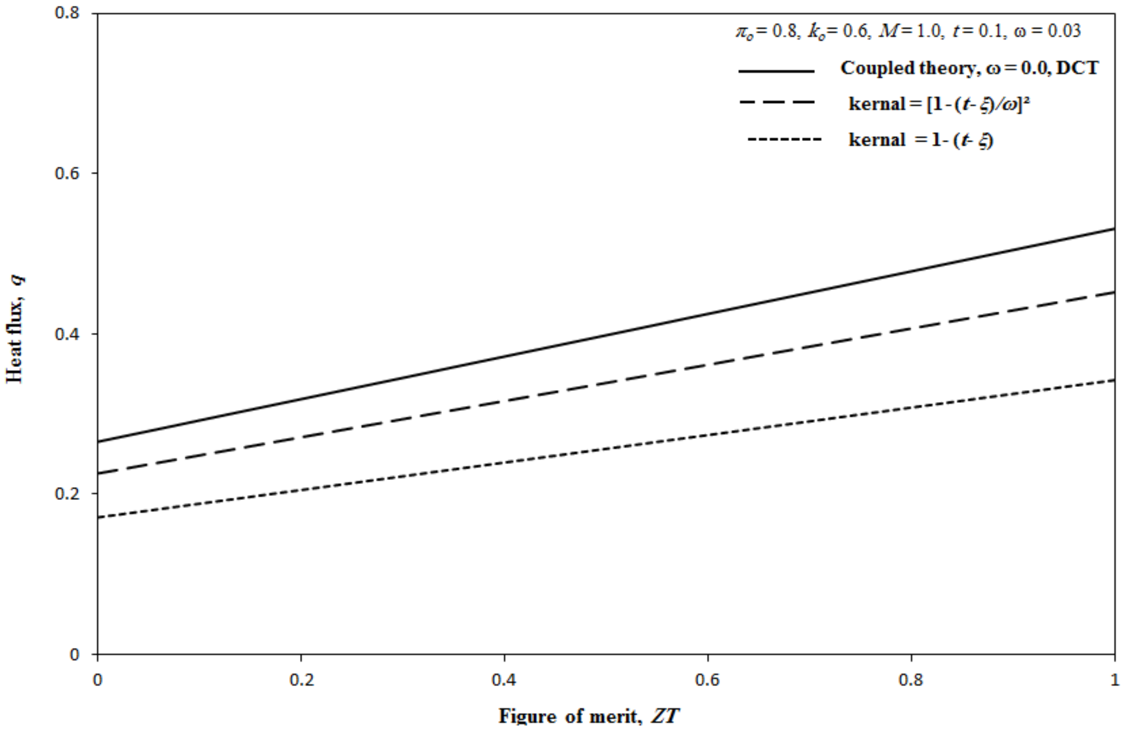

Fig. 2. The variation of heat flux q versus figure of merit ZT for different forms of kernal function $K(t, \xi)$

Figure 3 exhibits the space variation of the temperature distribution. In this figure, the solid line represents the solution obtained in the frame of dynamic coupled theory (Biot theory, $\omega = 0$ [1]) and the other lines represent the solutions obtained in the case $\omega > 0$. We observed that the temperature fields have been influenced when delay ω, where the expanding of the estimation of the parameter causes diminishing in temperature fields. The warm waves are consistent capacities, smooth, and reach to unfaltering state contingent upon the estimation of time-delay ω, which implies that the particles transport the warmth to different particles effectively and this makes the diminishing rate of the temperature more noteworthy than different ones. Additionally, the warm waves cut the *x*-hub all the more quickly when increments.

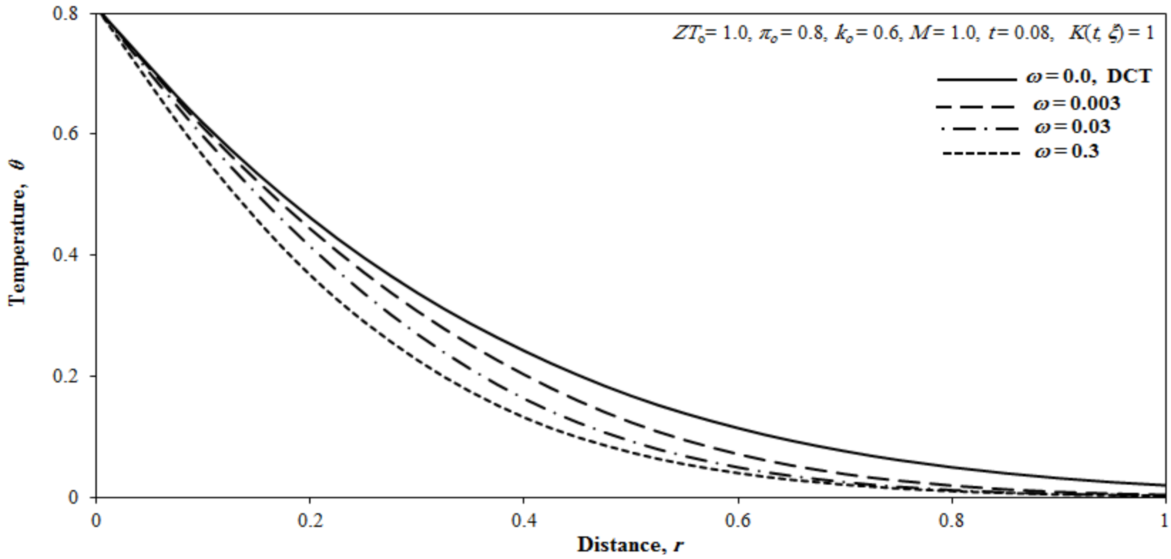

Fig. 3. The variation of temperature vs. distance for different values of time-delay ω

Figures 4, 5, and 6 show the variety of temperature, displacement, and stress circulations in thermoelectric circular depression with spiral separation r for three values of figure-of-merit at room temperature ZT_o, namely, ZT_o = 1, 3 and 5. We noticed that the stress and displacement field has been affected by the figure-of-merit values, where the expanding of the estimation of figure-of-merit causes decreasing in the magnitude of the stress and displacement field while causes increasing in the temperature.

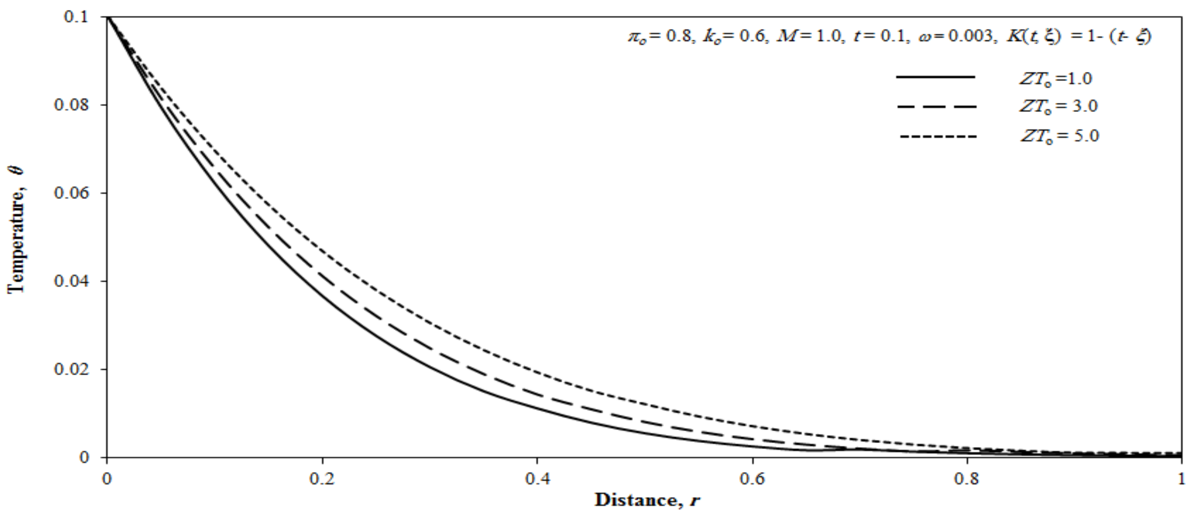

Fig. 4. The variation of temperature vs. distance for different values of figure-of-merit at room temperature ZT_o

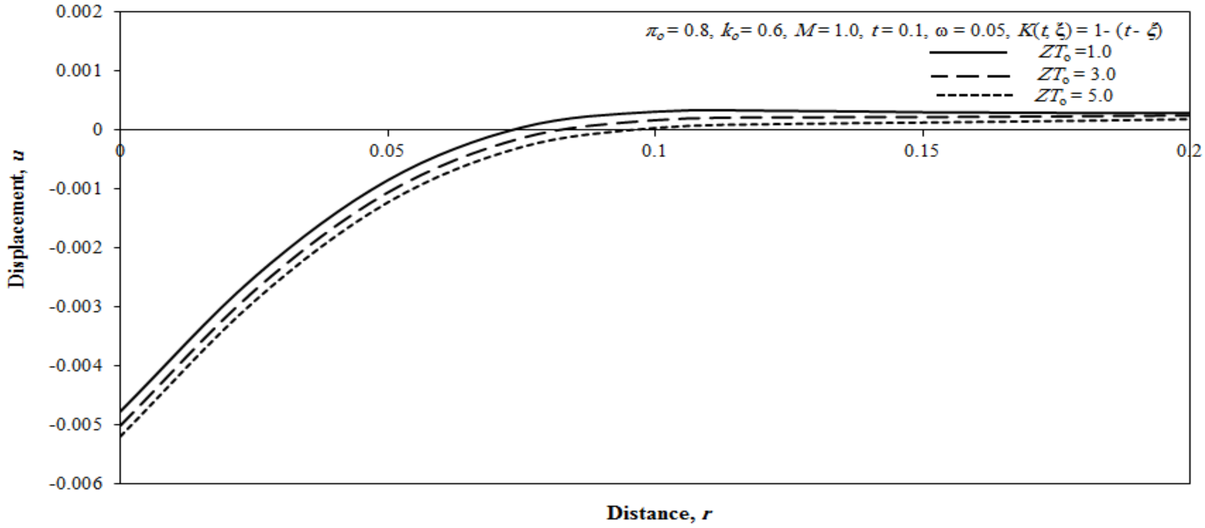

Fig. 5. The variation of displacement vs. distance for different values of figur-of-miret at room temperature ZT_o

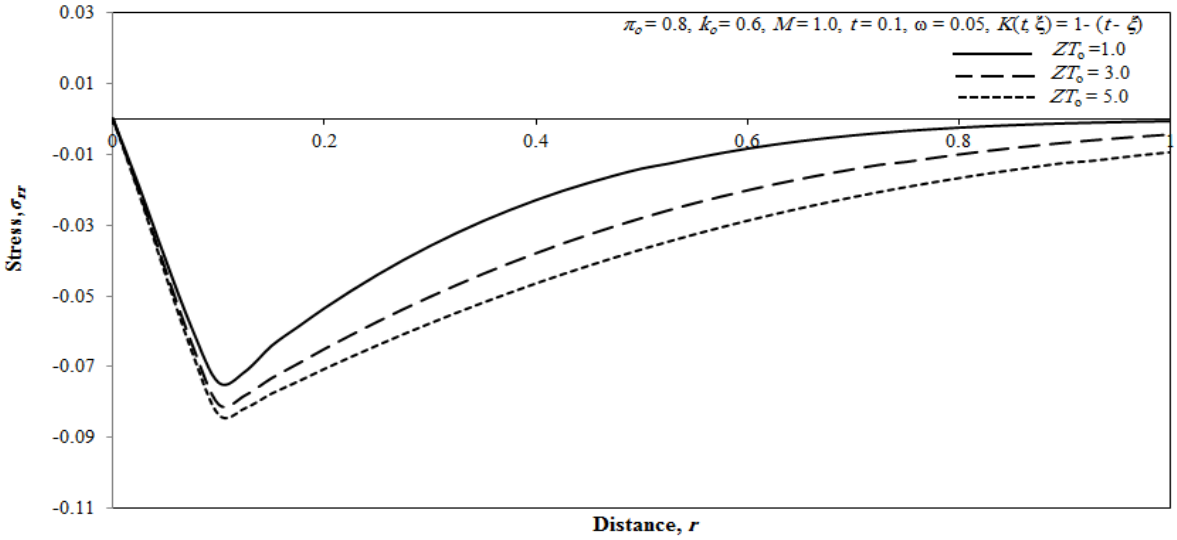

Fig. 6. The variation of stress vs. distance for different values of figur-of-miret at room temperature ZT_o

Figures 7 and 8 display the displacement and stress distributions with distance for two different theories; Biot theory, $\omega = 0$ and MDD theory, $\omega > 0$ when the magnetic number has two values M ($M = 0$, absent of the magnetic field and $M > 0$ in the presence of the magnetic field). We find that the attractive field acts to diminish the displacement and stress fields. This is generally known as attractive damping. It is possible to compare the results with those for the generalized thermoelasticity theory [44,45] and generalized thermo-viscoelasticity theory [46]. It was found that thermoelectric viscoelasticity's memory-dependent theory predicts a value for less than the generalized theory predicts. On the other hand, the stress value in the memory-dependent theory is greater.

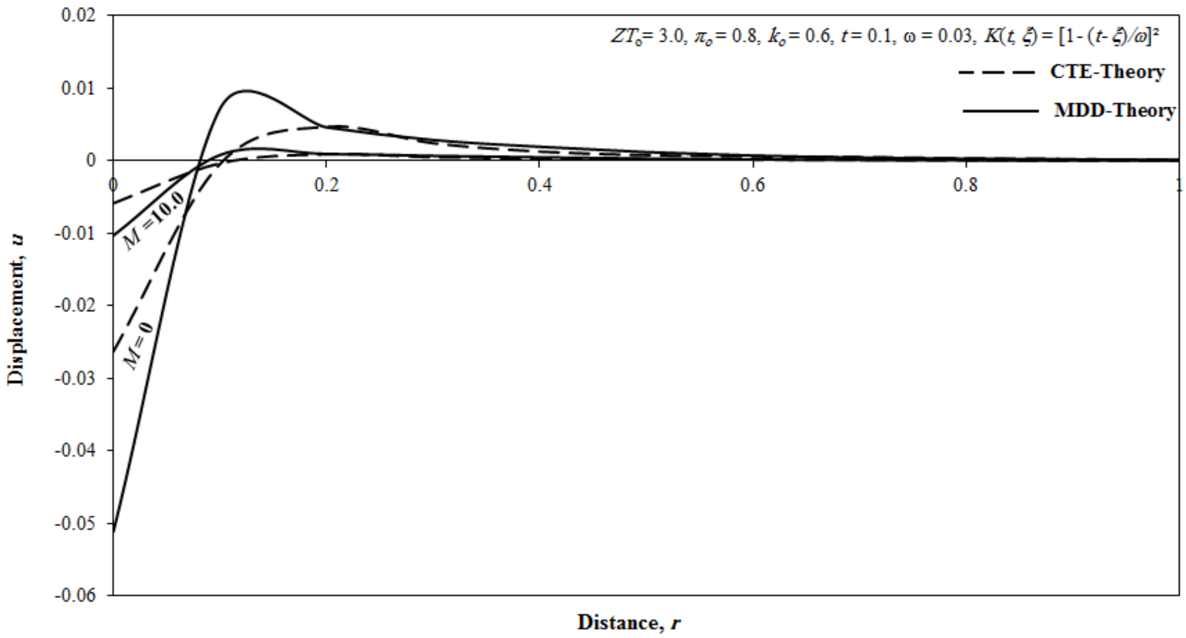

Fig. 7. The variation of displacement vs. distance for different values of magnetic number M

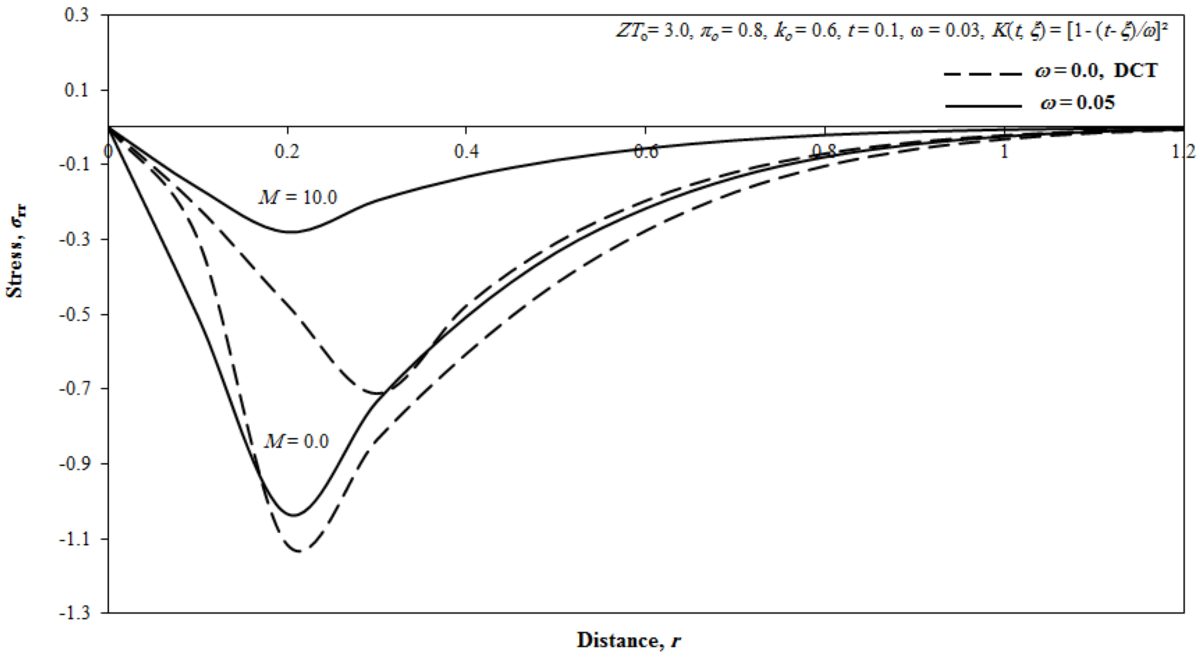

Fig. 8. The variation of stress component vs. distance for different values of magnetic number M

6. Conclusions

- The main goal of this work is to introduce a new mathematical model for the Fourier law of heat conduction with memory-dependent derivative and include the thermoelectric figure-of-merit. According to this new theory, we have to construct a new classification for materials according to a time-delay and kernel function where these variables become the new indicator of its ability to conduct heat in conducting medium. The certain issues of warm excitations in the hypothesis of coupled fields have a place with the electro-thermo-viscoelasticity. The expanding wide use in detecting and activation has pulled in much consideration towards hypotheses about materials displaying couplings between versatile, electric, attractive, and thermal fields. This model is more intuitionistic for understanding the physical significance and the comparing memory-dependent derivative condition is progressively expressive. The conditions of wave hypothesis of electro-thermo-viscoelasticity exposed to MDD based on the change of the Fourier law was built rough phenomenological conditions of thermo-electromagnetic versatility described by a limited speed of engendering of electromagnetic and flexible excitations. From the considered model we can set up some fundamental hypotheses on the straight coupled and generalized speculations of electro-thermo-viscoelasticity; for example, the coupled hypothesis $(\omega = 0)$ and the generalized case hypothesis $(\omega > 0)$. As per the after-effects of the work, we can see the nearness of MDD's parameters in Fourier law of warmth conduction can assume a crucial job in expanding or diminishing the speed of the wave proliferation of all fields through the thermoelectric medium. This model is more intuitionistic for understanding the physical significance and the comparing memory-dependent derivative condition is progressively expressive.
- This model enables us to improve the efficiency of a thermoelectric material figure-of-merit *ZT*. For a material to be a good thermoelectric cooler, it must have a high thermoelectric figure of merit, *ZT*. The result provides a motivation to investigate conducting thermoelectric materials as a new class of applicable thermoelectric materials [47]. The efficiency of a thermoelectric figure-of-merit is proportional to the temperature of the material particles.
- Owing to the complicated nature of the governing equations for the generalized thermo-viscoelasticity, few attempts have made to solve different problems in this field. These attempts utilized an approximate method valid for only a specific range of some parameters [48]. In this work, a simple method is introduced in the field of generalized thermoelectric viscoelasticity with memory-dependent derivative heat transfer and applied to the one-dimensional problem for a viscoelastic spherical cavity. This method gives exact solutions in the Laplace transform domain without any assumed restrictions on either the temperature or the displacement distributions. A numerical method based on a Fourier-series expansion has been used for the inversion process.
- Representative results for all functions for generalized theory are distinctly different from those obtained for the coupled theory. This due to the fact that thermal waves in the coupled theory travel with an infinite speed of propagation as opposed to finite speed in the generalized case. It is clear that for small values of time the solution is localized in a finite region. This region grows with increasing time and its edge is the location of the wavefront. This region is determined by the values of time t and time-delay. The predictions of the new theory are discussed and compared with dynamic classical coupled theory.
- The advantage of the considered a new model consists in:
  i) The discontinuities in temperature distribution disappeared.
  ii) The negative values of temperature that usually appear in the generalized theories of thermoelasticity vanished.

iii) The Kernel functions and time-delay of memory-dependent derivative can be arbitrarily chosen freely according to the necessity of applications [25].

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Compliance with ethical standards. *Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.*

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Nomenclature

λ, μ	Lame' constants
ρ	mass density
t	time
T	absolute temperature
θ	$= T - T_0$, such that $\lvert\theta / T_0\rvert << 1$
ε_{ij}	components of strain tensor
e	$= \varepsilon_{ii}$, dilatation
e_{ij}	components of strain deviator tensor
σ_{ij}	components of stress tensor
k_o	Seebeck coefficient
π_o	Peltier coefficient
k	thermal conductivity
C_E	specific heat at constant strains
K_o	$= \lambda + (2/3)\mu$, bulk modulus
c_o^2	$= \dfrac{K}{\rho}$, longitudinal wave speed
α_T	coefficient of linear thermal expansion
γ	$= 3K\alpha_T$
T_o	reference temperature
c_o	$= [(\lambda + 2\mu)/\rho]^{1/2}$, speed of propagation of isothermal elastic waves
η_o	$= \rho C_E / \kappa$
ε	$= \dfrac{\gamma^2 T_o}{k\,\eta_o \rho c_o^2}$, Thermal coupling parameter
Z	thermoelectric figure-of-merit
$\Gamma(.)$	Gamma function
A^*, β^*, α^*	empirical constants