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# Study of different theories of thermoelasticity under the Rayleigh wave propagation along an isothermal boundary

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**ABSTRACT**

The propagation of Rayleigh surface waves in an isotropic thermoelastic solid half-space is the focus of the current study, which takes into account the compact form of six distinct thermoelasticity theories. An isothermal boundary surface in the absence of tangential and normal stress is used to solve the problem. A dispersion equation with irrational terms is obtained after creating a mathematical model. This equation needs to be transformed into a rational polynomial equation in order to use the algebraic method to find exact complex roots. The roots are filters for in-homogenous wave propagation that decays with depth. Then these roots are used to compute the numerically characteristic properties of the Rayleigh wave, which include phase velocity, attenuation coefficient, and polarisation of particles. The results are presented graphically for particular cases of thermoelasticity by using the physical data of copper metal.

**KEYWORDS**

coupled model • G-N model • three phase lag model • L-S model • phase velocity • dual phase lag model G-L model • attenuation coefficient

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## Introduction

Both natural and artificial materials are rated according to how they react to wave propagation parameters including wave polarization (particle oscillations) and travel time spans (or phase velocities). Among other things, specific heat, thermal expansion, and thermal conductivity all have an impact on these quantifiable values. The relationship between an elastic material temperature and the distribution of strain and stress, as well as the reciprocal impact of induced deformation on temperature distribution, are all examined by thermoelasticity theory. Applications for wave propagation phenomena are widely used in seismology, oil exploration, mineral and geophysical exploration. Thermoelasticity plane wave propagation has numerous applications in diverse engineering domains. When researching many facets of an earthquake, the surface waves are a great resource.

Biot [1] established the coupled theory of thermoelasticity using hyperbolic-parabolic field equations. Lord and Shulman (L-S) [2] and Green and Lindsay(G-L) [3] extended the coupled theory and called as generalised thermoelasticity. Green and Naghdi (G-N) [4] created a thermoelasticity theory lacking of energy dissipation. These theories [2–4] admit a finite heat-propagation speed, then the difference from coupled theory is created. Hetnarski and Ignaczak [5] and Ignaczak and Ostoja-Starzewski [6] examined these illustrative generalised thermoelasticity theories. Tzou [7] worked on the two-phase- lag model. Roy-Choudhuri [8] studied the three-phase lag model. Numerous scholars have looked into wave propagation problems associated with coupled or generalised thermoelasticity [9–15].

The surface waves that travel along an elastic solid medium's free surface were examined by Rayleigh [16]. These Rayleigh waves are employed to observe the mechanical and structural characteristics of any material since they may travel over the surface, penetrate thick solid materials to a depth of one wavelength, and exhibit extremely sensitive behaviour to surface flaws. In thermoelasticity, the Rayleigh-type surface waves are thought to be useful in a variety of engineering domains and emerging technologies. There have been numerous reported uses of the Rayleigh wave in thermoelasticity theory up to this point. Some of them are as follows: Sinha and Sinha [17] examined the impact of modulating the Rayleigh wave velocity on altering the temperature environment. Sharma [18] investigated the characteristics of Rayleigh waves by employing the functional iteration approach to solve a difficult transcendental problem. Tomita and Shindo [19] analysed how Rayleigh waves propagate in a fully conducting elastic half-space in response to magnetic fields. Chadwick and Windle [20] examined the consequences of Rayleigh wave propagation along insulated and isothermal boundaries.

Dawn and Chakraborty [21] studied about the Green and Lindsay hypothesis in relation to the Rayleigh wave investigation in thermoelastic media. Chadwick and Windle [20] presented the work on the properties of the Rayleigh wave by considering two different boundary conditions. Ahmed [22] examined how temperature stress affected Rayleigh wave propagation in a granular media. Sharma et al. [23] analysed how thermal relaxation and rotation affected the Rayleigh surface waves in piezothermoelastic half-space. Abd-alla [24] studied the surface wave in a generalised thermoelastic medium in the relaxation of the thermal time effect. Mahmoud [25] investigated the effect of magnetic field, rotation, relaxation times, gravity field, and initial stress on Rayleigh wave velocity in the space of a granular medium. Noguchi et al. [26] examined the use of Rayleigh waves by taking into account how long-period ground motion is affected by earthquakes. Bucur et al. [27] examined the ways in which thermal fields in a linear thermoelastic material with voids can dampen Rayleigh waves and harmonic waves. Zhao et al. [28] An analytical approach to the seismic response of composite-lined tunnels under Rayleigh waves in elastic ground. Xinxin et al. [29] studied seismic Rayleigh wave imaging for oil exploration using genetic-damped least squares joint inversion and multi-channel surface wave analysis. Singh and Verma [30] examined many theories of thermoelasticity to study the Rayleigh wave's propagation in thermoelastic solid half-space. Kumar et al. [31] examined the L-S theory under the influence of the initial stress, magnetic field, two temperatures, and diffusion in order to study the fundamental equation of thermoelasticity. Kumar and Gupta [32] examined Rayleigh waves in a mass-diffusing, generalised thermoelastic media. Kumar et al. [33] examined the microrotation, tangential component of displacement, and normal boundary conditions for the shear, normal, and shear couple tractions at the surfaces, respectively.

Haque and Biswas [34] investigated the wave propagation using an algebraic differential equations Eigen value. Determine the attenuation coefficient and phase velocity, then compare the graphs for void and non-void space. Saeed et al. [35] examined the propagation of Rayleigh waves in a temperature-dependent semi-conductor thermoelastic material. Singh and Kashyap [36] the propagation of micro polar thermoelastic material with time delay under memory-dependent derivatives has been investigated.

The current paper is structured as follows: in the "Basic equations" section, we address the compact version of the heat conduction equation of the six theories of thermoelasticity after first providing the basic equations employed in the study. The problem is formulated in the "Formulation of problem" section by utilising the displacement vector in the form of a potential function. Using potential functions, determine the longitudinal and transversal wave velocities in the "Solution of problem" section. In the "Boundary surface conditions" section, the secular dispersion equation is created using boundary conditions in order to discover complex filtered roots, which are used to determine the phase velocity, attenuation coefficient, and particle path, among other characteristics of the Rayleigh wave. In the "Path of particle" section, the phase velocity, attenuation coefficient, and path of six peculiar examples are presented graphically.

## Basic equations

A compact form of the equations for thermoelasticity theories in the absence of external heat sources is as follows, in accordance with Kumar and Gupta [37].

The temperature-stress-strain relationship: $\sigma_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} - \delta_{ij}\beta\left(1+\tau_1\frac{\partial}{\partial t}\right)T.$ (1)

Relation between strain and displacement: $e_{13} = e_{31} = (\partial u_1/\partial x_3 + \partial u_3/\partial x_1)/2.$ (2)

The equations of motion: $(\lambda+\mu)u_{k,ki} + \mu u_{i,kk} - \beta\left(1+\tau_1\frac{\partial}{\partial t}\right)T_{,i} = \rho\ddot{u}_i.$ (3)

Modified Fourier's law: $K'\left(m^* + t_1\frac{\partial}{\partial t} + t_3\frac{\partial^2}{\partial t^2}\right)T_{,i} = -q_i.$ (4)

Energy equation: $\rho T_0\dot{S} = -q_{i,i}.$ (5)

Entropy-strain-temperature relation:

$$\rho C_E\left(m_1\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2} + t_2\frac{\partial^3}{\partial t^3} + t_4\frac{\partial^4}{\partial t^4}\right)T + T_0\beta\left(m_1\frac{\partial}{\partial t} + m_0\tau_0\frac{\partial^2}{\partial t^2} + t_2\frac{\partial^3}{\partial t^3} + t_4\frac{\partial^4}{\partial t^4}\right)e_{kk} = \rho T_0\dot{S}. \quad (6)$$

The heat conduction equation:

$$K'\left(m^* + t_1\frac{\partial}{\partial t} + t_3\frac{\partial^2}{\partial t^2}\right)T_{,ii} = \rho\, C_E\left(m_1\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2} + t_2\frac{\partial^3}{\partial t^3} + t_4\frac{\partial^4}{\partial t^4}\right)T + \\ +T_0\beta\left(m_1\frac{\partial}{\partial t} + m_0\tau_0\frac{\partial^2}{\partial t^2} + t_2\frac{\partial^3}{\partial t^3} + t_4\frac{\partial^4}{\partial t^4}\right)e_{kk}. \quad (7)$$

In Eqs. (1)–(7), constants and parameters are used as following: $\lambda$, $\mu$ are the lame's constants, $\rho$ is a mass density, $C_E$a the specific heat at the constant strain, $e_{kk}$ are an dilatation, $q_i$are the heat flux components, $u_i$ are the displacement components, $\sigma_{ij}$ are the stress tensor components, $S$ is an entropy per unit mass, $K'$ is the thermal conductivity, $T$ is an increment in temperature, $T = \theta - T_0$, where $T_0$ is a reference temperature and $\theta$ is an absolute temperature with condition satisfied as $|T/T_0| << 1$, $\beta = (3\lambda + 2\mu)\alpha_t$, $\alpha_t$ is the coefficient of thermal linear expansion, $\tau_0$, $\tau_1$, $\tau_q$, $\tau_T$, $\tau_v$ are relaxation times in thermal with condition $\tau_1 \geq \tau_0 \geq 0$, phase lags of heat flux, temperature gradient and thermal displacement gradient respectively, where $m^*$, $m_0$, $m_1$, $t_1, t_3$, $t_2$, $t_4$, $\tau_0$, $\tau_1$, are parameters.

Rayleigh surface wave propagation through an isotropic medium with thermoelasticity. The following values of the parameters in Eqs. (3) and (7) are used to study the various theories:

1. Coupled theory (C-T) condition of thermoelasticity is obtained when:

$$m^* = \mathrm{m}_1 = 1,\ m_0 = t_1 = t_2 = t_3 = t_4 = \tau_0 = \tau_1 = 0, \tag{8}$$

2. The thermoelasticity theory known as Lord-Shulman (L-S) is derived when:

$$m^* = m_0 = m_1 = 1, t_1 = t_2 = t_3 = t_4 = \tau_1 = 0, \tag{9}$$

3. In thermoelasticity, the Green- Lindsay (G-L) theory is derived when:

$$m^* = m_1 = 1,\ m_0 = t_1 = t_2 = t_3 = t_4 = 0, \tag{10}$$

4. The thermoelasticity theory known as Green-Nagdhi (Type-III) or G-N is derived from:

$$m^* > 0, \mathrm{m}_0 = \tau_0 = t_1 = 1, \mathrm{m}_1 = t_2 = t_3 = t_4 = \tau_1 = 0, \tag{11}$$

put in (7) results into equation:

$$K'\left(m^* + \frac{\partial}{\partial t}\right)T_{,ii} = \rho C_E \ddot{T} + \beta T_0 \ddot{e}_{kk}. \tag{12}$$

Here $m^*$ is a constant having a dimension 1/sec, $\dot{T} = \vartheta$, and $m^* K' = K'^*$ is a constant characteristic of the theory. Equation (12) become:

$$K'^* T_{,ii} + K' \vartheta_{,ii} = \rho C_E \ddot{T} + \beta T_0 \ddot{e}_{kk}, \tag{13}$$

Subcase: when $K' = 0$ in Eq. (13), Green- Nagdhi (Type-II) theory is obtained.

5. The thermoelasticity two-phase-lag theory is derived when:

$$m^* = 1, \mathrm{m}_0 = m_1 = 1, \tau_1 = t_3 = t_4 = 0, t_1 = \tau_T,\ t_2 = \frac{\tau_q^2}{2}, \tau_0 = \tau_q, \tag{14}$$

6. The thermoelasticity three-phase-lag theory is derived when:

$$m_0 = \tau_0 = 1, \mathrm{m}_1 = \tau_1 = 0, t_2 = \tau_q, t_1 = 1 + m^* \tau_v, t_3 = \tau_T, t_4 = \frac{\tau_q^2}{2}. \tag{15}$$

## Formulation of problem

To solve the problem related to two-dimensional space, consider the displacement vector $u = (u_1, 0, u_3)$ in medium. The dimensionless quantities listed below are defined as:

$$T' = \frac{\beta T}{\rho c_1^2},\ \{x_i', u_i'\} = \left\{\frac{\omega_1^* x_i}{C_1}, \frac{\omega_1^* u_i}{C_1}\right\},\ \nabla^2 = \frac{\partial^2}{\partial x_i^2},\ i = 1, 3,$$

$$\{\tau_1', \tau_0'\} = \{\tau_1, \tau_0\}\omega_1^*, t' = \omega_1^* t,\ \{\tau_q', \tau_T', \tau_v'\} = \{\tau_q, \tau_T, \tau_v\}\omega_1^*, \tag{16}$$

$$\sigma_{ij}' = \frac{\sigma_{ij}}{\beta T_0}, \omega_1^* = \frac{\rho C_E C_1^2}{K'},\ \delta^2 = \frac{C_2^2}{C_1^2},\ \ C_1^2 = \frac{\lambda + 2\mu}{\rho},\ \ C_2^2 = \frac{\mu}{\rho}.$$

The displacement components in form of potential function $\phi_1$, $\phi_2$, $\phi_3$ can be written as:

$$u_1 = \frac{\partial \phi_i}{\partial x_1} + \frac{\partial \phi_3}{\partial x_3},\ u_3 = \frac{\partial \phi_i}{\partial x_3} - \frac{\partial \phi_3}{\partial x_1},\ i = 1,\ 2. \tag{17}$$

In Eqs. (3) and (7) with the help of (16), after suppressing the primes, apply the Eq. (17) we obtain:

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\varphi - \left(1 + \tau_1 \frac{\partial}{\partial t}\right)T = 0, \tag{18}$$

$$\nabla^2 \varphi_3 - \frac{1}{\delta^2}\frac{\partial^2 \varphi_3}{\partial t^2} = 0, \tag{19}$$

$$\left(m^* + t_1 \frac{\partial}{\partial t} + t_3 \frac{\partial^2}{\partial t^2}\right)\nabla^2 T = \tag{20}$$

$$= \left(m_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} + t_2 \frac{\partial^3}{\partial t^3} + t_4 \frac{\partial^4}{\partial t^4}\right)T + \frac{\beta^2 T_0}{\rho^2 C_E C_1^2}\left(m_1 \frac{\partial}{\partial t} + m_0 \tau_0 \frac{\partial^2}{\partial t^2} + t_2 \frac{\partial^3}{\partial t^3} + t_4 \frac{\partial^4}{\partial t^4}\right)\nabla^2 \varphi.$$

### Solution of problem

We assume, for propagation of harmonic wave in consider plane as:
$$\{\phi,\ \phi_3,\ T\}(x_1,\ x_3,\ t) = \{\bar{\phi},\bar{\phi}_3,\bar{T}\}e^{-i\omega t}. \tag{21}$$
Substitute Eq. (21) in Eqs. (18) and (20) and then simplified, we get:
$$(Y_1\nabla^4 + Y_2\nabla^2 + Y_3)\bar{\varphi} = 0 \tag{22}$$
where $Y_1 = R_2, Y_2 = R_2\omega^2 - R_3 - R_1R_4, Y_3 = -\omega^2R_3, R_1 = 1 - i\omega\tau_1, R_2 = (m^* - it_1\omega - t_3\omega^2)$, $R_3 = (-im_1\omega - \tau_0\omega^2 + it_2\omega^3 + t_4\omega^4),\ R_4 = \left(\frac{\beta^2T_0}{\rho^2C_EC_1^2}\right)(-im_1\omega - \tau_0m_0\omega^2 + it_2\omega^3 + t_4\omega^4)$.

General solution $\bar{\phi}$ can be written as:
$$\bar{\phi} = \bar{\phi}_1 + \bar{\phi}_2, \tag{23}$$
where the potential $\bar{\varphi}_1,\ \bar{\varphi}_2$ are solutions of equation given by:
$$\left[\nabla^2 + \frac{\omega^2}{V_i^2}\right]\bar{\varphi}_i = 0, \quad i = 1,\ \ 2 \tag{24}$$
where $V_1,\ V_2$, are the velocities of longitudinal waves ($P$and $SV$ wave), are the roots of equation:
$$R_3V^4 - R_2\omega^2V^2 + R_1\omega^4 = 0. \tag{25}$$
Equation (19) can be solved for the transverse wave velocity $V_3 = \delta$ using Eq. (21). The result is provided by:
$$\left[\nabla^2 + \frac{\omega^2}{V_3^2}\right]\bar{\varphi}_3 = 0, \tag{26}$$
By using Eqs. (7), (21), (23) and (24) we obtain:
$$\{\varphi, T\} = \textstyle\sum_{k=1}^{2}\{1, n_k\}\varphi_k, \tag{27}$$
where $n_k = \frac{S_1S_4\omega^2}{S_1S_2\omega^2 - S_3{V_k}^2},\quad k = 1,\ 2.$

The displacement potentials $\varphi_1,\ \varphi_2,\ \varphi_3$ for the propagation of harmonic wave with exponential decay in a plane is given as:
$$\varphi_k = A_ke^{i\omega\left(\frac{x_1 + q_kx_3}{c} - t\right)},\quad k = 1,\ 2,\ 3, \tag{28}$$
where apparent phase velocity used as $c$, $q_1 = \sqrt{\frac{c^2}{{V_1}^2} - 1},\quad q_2\ \sqrt{\frac{c^2}{{V_2}^2} - 1}\ ,\quad q_3 = \sqrt{\frac{c^2}{\delta^2} - 1}.$

## Boundary surface conditions

For the isothermal and stress-free surface $x_3 = 0$:
(a) vanish normal and tangential stress component:
$$\sigma_{33} = 0, \tag{29}$$
$$\sigma_{31} = 0, \tag{30}$$
(b) isothermal boundary surface:
$$T = 0, \tag{31}$$
Equation (28) is used in Eqs. (29)–(31) with the aid of Eqs. (1), (2), (17), and (27) to get the following system of three homogeneous equations:
$$\textstyle\sum_{k=1}^{3} c_{ik}\,A_k = 0, \tag{32}$$
where $c_{11} = -\omega^2\Pi_1 + 2\frac{\omega^2}{h} - \beta\frac{n_1}{\rho},\ c_{21} = 2q_1,\ c_{31} = n_1, c_{12} = -\omega^2\Pi_2 + 2\frac{\omega^2}{h} - \beta\frac{n_2}{\rho}$, $c_{22} = 2q_2,\ c_{32} = n_2, c_{13} = 2\frac{\omega^2q_3}{h},\ c_{23} = q_3^2 - 1,\ c_{33} = 0.$

The homogeneous system of equations has a non-trivial solution when the determinant of the coefficients of the equations in Eq. (32) disappears:

$$(2-h)[(2-\Pi_1 h)-\eta(2-\Pi_2 h)]=-4(q_1-\eta q_2)q_3, \tag{33}$$

where $\eta=\frac{n_1}{n_2}$, $h=\frac{c^2}{\delta^2}$, $\Pi_i=\frac{\lambda+2\mu}{\rho V_i^2}$, $i=1,\ 2$.

Since some of the terms in Eq. (32) are irrational, an algebraic approach cannot be used to solve it. This equation can be solved by eliminating radicals using two squares and manipulation, when applied to the given equation, yields an algebraic equation of degree seven that may be written as:

$$\sum_{k=0}^{7} c_k\, h^k=0, \tag{34}$$

where $c_0=1024\eta(d-\eta\varepsilon_s)$, $c_4=-32a[a(d-2\eta)+(a+b)g]$, $c_1=256[d^2-\eta(g+4d)]-$ $-1024\eta^2(\varepsilon_p+2\varepsilon_s)$, $c_5=8a^2(3g+4ab-8d)$, $c_2=128[2\eta a(a+b)-(d--2\eta)g]+$ $+1024\eta^2(2\varepsilon_p+\varepsilon_s)$, $c_6=-8a^3(a+b)$, $c_3=16[g^2+8a(a+b)(d-2\eta)-4\eta a^2]-1024\eta^2\varepsilon_p$, $c_7=a^4$, $a=(\Pi_1-\eta\Pi_2)$, $b=1-\eta$, $d=ab-(\varepsilon_1+\eta^2\varepsilon_2)$, $g=a^2+b^2+4d$, $\varepsilon_p=\varepsilon_1\varepsilon_2$, $\varepsilon_s=\varepsilon_1+\varepsilon_2$, $\varepsilon_i=\left(\frac{\delta}{V_i}\right)^2$, $i=1,\ 2$.

Seven complex roots are obtained from an algebraic Eq. (34) some of which are added after radicals are eliminated from Eq. (33). When the initial dispersion equation is not satisfied, these roots are recognised and distinguished in Eq. (33). The remaining roots that satisfy Eq. (33) are taken into consideration for the wave field decay as the increases of $x_3$ in the medium. These roots of the Rayleigh wave Eqs. (33) and (34) describing the existence and propagation in the isothermal plane boundary of a thermoelastic material are satisfied. The coefficient $c_i$ depend upon $\omega$ therefore, the phase velocity $V$ calculated from a root of Eq. (34) is also a function of $\omega$. This implies the dispersive behaviour of Rayleigh wave on isothermal surface in thermoelastic medium. The complex value of $c$ shows that waves are attenuated and positive imaginary part of vertical slowness $(q_i/c)$, $i=1,\ 2,\ 3$ indicates decay of wave with depth in region $x_3\geq 0$.

Following [38], the phase velocity $V$ and attenuation coefficient $Q^{-1}$ can be calculated as:

$$V=\frac{\delta|h|}{Re(\sqrt{h})}=\frac{|c^2|}{Re(c)}, \tag{35}$$

$$Q^{-1}=-\frac{Im(h)}{Re(h)}=\frac{Im(1/c^2)}{Re(1/c^2)}. \tag{36}$$

Path of surface particles:

$$\varphi_i=A_1\lambda_i e^{i(kx_1-\omega t)+ikx_3q_i},\quad i=1,\ 2,\ 3, \tag{37}$$

where $\lambda_i=A_i/A_1$, $i=1,\ 2,\ 3$ are solution of Eq. (30) and $k=\omega/c$ is the complex number, $\lambda_1=1$, $\lambda_2=-\eta$, $\lambda_3=2[(\eta q_2-q_1)/(q_3^2-1)]$.

By using Eq. (37) in Eqs. (17) and (23), we get:

$$(u_1,u_3)=\left(|U_0|e^{i\arg U_0},|W_0|e^{i\arg W_0}\right)e^{i(kx_1-\omega t)}, \tag{38}$$

$$\begin{pmatrix}U_0\\ W_0\end{pmatrix}=\begin{pmatrix} i\left[\lambda_1 e^{ik_Rx_3\delta_1}+\lambda_2 e^{ik_Rx_3\delta_2}+\lambda_3 q_3 e^{ik_Rx_3\delta_3}\right]\\ \left[\lambda_1 q_1 e^{ik_Rx_3\delta_1}+\lambda_2 q_2 e^{ik_Rx_3\delta_2}-\lambda_3 e^{ik_Rx_3\delta_3}\right]\end{pmatrix}kA_1e^{i(kx_1-\omega t)}, \tag{39}$$

where $\delta_i=[1-i(c_I/c_R)]q_i$, $i=1,\ 2,\ 3$, $R$ is used for real part and $I$ is used for imaginary part of complex quantity, $K$ is wave number.

By using Eqs. (37) and (27), we get:

$$T=|T_0|e^{i\arg T_0}e^{i(kx_1-\omega t)}, \tag{40}$$

$$T_0=A_1\left(n_1\lambda_1 e^{ik_Rx_3\delta_1}+n_2\lambda_2 e^{ik_Rx_3\delta_2}\right). \tag{41}$$

On the boundary surface $x_3=0$ Eq. (38) by considering the real part we get:

$$\left.\begin{aligned} U &= |U_0|\,cos(arg\,U_0 + \Phi)\,e^{-k_I x_1} \\ W &= |W_0|\,sin(arg\,W_0 + \Phi)\,e^{-k_I x_1} \end{aligned}\right\}, \tag{42}$$

where $\Phi = K_R x - \omega t$ parameter varies in $0.2\pi$ to show the path traced. By using Eq. (28) in parametric form traces elliptical path.

### Special Cases

Case 1: In order to derive the secular equation for the thermoelasticity coupled theory. Substitute $m^* = m_1 = 1,\ m_0 = t_1 = t_2 = t_3 = t_4 = \tau_0 = \tau_1 = 0$, into Eq. (25).

For $\Pi_1 = 0,\ \eta = 0$ Rayleigh wave propagation in an elastic medium is represented by Eq. (33) when reduced to equation $(2-h)^2 = -4q_1q_3$. In line with Ewing et al. [39].

Case 2: In order to derive the secular equation for the thermoelasticity L-S theory with one relaxation of times. Substitute $m^* = m_0 = m_1 = 1, t_1 = t_2 = t_3 = t_4 = \tau_1 = 0$, into Eq. (25).

Case 3: In order to derive the secular equation for the thermoelasticity G-L theory with two relaxations of times. Substitute $m^* = m_1 = 1,\ m_0 = t_1 = t_2 = t_3 = t_4 = 0$, into Eq. (25).

Case 4: In order to derive the secular equation for the thermoelasticity G-N (Type-III) theory. Substitute $m^* > 0,\ \ m_0 = \tau_0 = t_1 = 1,\ \ m_1 = t_2 = t_3 = t_4 = \tau_1 = 0$, into Eq. (25).

Case 5: In order to derive the secular equation for the thermoelasticity two-phase -lag theory. Put $m^* = 1,\ m_0 = m_1 = 1, \tau_1 = t_3 = t_4 = 0, t_1 = \tau_T,\ t_2 = \tau_q^2/2, \tau_0 = \tau_q$, into Eq. (25).

Case 6: In order to derive the secular equation for the thermoelasticity three-phase -lag theory. Put $m_0 = \tau_0 = 1,\ m_1 = \tau_1 = 0, t_2 = \tau_q, t_1 = 1 + m^*\tau_v, t_3 = \tau_T, t_4 = \tau_q^2/2$, into Eq. (25).

## Numerical results and Discussion

### Findings of phase velocity, attenuation coefficients

Phase velocity and attenuation coefficient for various theories of thermoelasticity are compared using MATLAB software, adopting frequencies ranging from 0.1 to 59.1 Hz. Now we find numerical results for copper material [38] the required data is given by:
$K = 400\ \mathrm{Wm^{-1}},\ \ \lambda = 77.6\ \mathrm{GPa},\ \ \mu = 38.6\ \mathrm{GPa},\ \ \alpha_t = 1.78 \times 10^{-5}\mathrm{K^{-1}}$,
$C_E = 383\ \mathrm{J\ Kg^{-1}/K},\ \ \rho = 8920\ \mathrm{kg/m^3},\ \ T_0 = 318\ \mathrm{K}$.

The relaxation time is given by: $\tau_0 = 0.1\ \mathrm{s}, \tau_q = 0.6\ \mathrm{s},\ \tau_1 = 0.2\ \mathrm{s},\ \tau_v = 0.5\ \mathrm{s},\ \tau_T = 0.4\ \mathrm{s},\ \ m^* = 0.38710\ \ \mathrm{sec^{-1}}$.

Tables 1 and 2 display the program's output for comparing various thermoelasticity theories with respect to frequency. The phase velocity and attenuation coefficient change with respect to frequency for various thermoelasticity theories are depicted in Fig. 1. The following thermoelasticity theories are represented in Fig. 1: coupled theory (C−T), Lord - Shulman theory (L-S), green - Lindsay theory (G-L), green - Naghdi theory (G-N), two-phase- lag theory (DPL), and three-phase-lag theory (TPL). Solid blue line, red dashed line, green dash line with asterisk, red dash line with circle, dark green only asterisk, and sky blue with plus sign, respectively. The phase velocity of the G-L theory increases smoothly in the range $0 \le \omega \le 5.1$, then starts decreasing with an increase in frequency and becomes stationary for $\omega \ge 55.1$. The phase velocity of the three-phase-lag theory increases smoothly in the range $0 \le \omega \le 7.1$, then starts decreasing with an increase in frequency and becomes stationary for $\omega \ge 55.1$.

**Table 1.** Phase velocity (V) value with respect to frequency in the context of various thermoelasticity theories

| Frequency $\omega$ | C-T ( V ) | L-S ( V ) | G-L ( V ) | G-N ( V ) | DPL ( V ) | TPL ( V ) |
|---|---|---|---|---|---|---|
| 0.1 | 0.4582 | 0.4581 | 0.457 | 0.4295 | 0.458 | 0.4295 |
| 1.1 | 0.4197 | 0.4216 | 0.4334 | 0.4257 | 0.4258 | 0.4249 |
| 2.1 | 0.4345 | 0.4374 | 0.4993 | 0.4793 | 0.4524 | 0.4955 |
| 3.1 | 0.4469 | 0.4506 | 0.4938 | 0.4776 | 0.4867 | 0.5028 |
| 4.1 | 0.4559 | 0.4603 | 0.4996 | 0.4816 | 0.4929 | 0.5098 |
| 5.1 | 0.4624 | 0.4675 | 0.5006 | 0.4848 | 0.4983 | 0.5154 |
| 6.1 | 0.4672 | 0.4729 | 0.4246 | 0.4873 | 0.5027 | 0.5205 |
| 7.1 | 0.471 | 0.4771 | 0.368 | 0.4894 | 0.5066 | 0.5252 |
| 8.1 | 0.474 | 0.4805 | 0.3244 | 0.4912 | 0.5101 | 0.4763 |
| 9.1 | 0.4764 | 0.4832 | 0.2899 | 0.4927 | 0.5133 | 0.4622 |
| 10.1 | 0.4784 | 0.4855 | 0.262 | 0.4941 | 0.5165 | 0.4427 |
| 11.1 | 0.48 | 0.4875 | 0.2389 | 0.4953 | 0.5195 | 0.4168 |
| 12.1 | 0.4814 | 0.4892 | 0.2195 | 0.4964 | 0.5225 | 0.3849 |
| 13.1 | 0.4826 | 0.4906 | 0.203 | 0.4975 | 0.5255 | 0.3495 |
| 14.1 | 0.4837 | 0.492 | 0.1887 | 0.4986 | 0.5285 | 0.3136 |
| 15.1 | 0.4846 | 0.4932 | 0.1764 | 0.4996 | 0.5314 | 0.2795 |
| 16.1 | 0.4854 | 0.4942 | 0.1655 | 0.5006 | 0.5343 | 0.2489 |
| 17.1 | 0.4861 | 0.4952 | 0.1559 | 0.5015 | 0.5372 | 0.2221 |
| 18.1 | 0.4868 | 0.4961 | 0.1474 | 0.5025 | 0.5401 | 0.1989 |
| 19.1 | 0.4874 | 0.497 | 0.1397 | 0.7187 | 0.5429 | 0.179 |
| 20.1 | 0.4879 | 0.4978 | 0.1328 | 0.6973 | 0.5458 | 0.1618 |
| 21.1 | 0.4884 | 0.4985 | 0.1265 | 0.6774 | 0.5485 | 0.1469 |
| 22.1 | 0.4888 | 0.4992 | 0.1208 | 0.6591 | 0.5513 | 0.134 |
| 23.1 | 0.4892 | 0.4999 | 0.1156 | 0.642 | 0.554 | 0.1226 |
| 24.1 | 0.4895 | 0.5006 | 0.1108 | 0.626 | 0.5567 | 0.1127 |
| 25.1 | 0.4899 | 0.5012 | 0.1064 | 0.611 | 0.5594 | 0.1039 |
| 26.1 | 0.4902 | 0.5018 | 0.1024 | 0.597 | 0.562 | 0.0961 |
| 27.1 | 0.4905 | 0.5023 | 0.0986 | 0.5838 | 0.7117 | 0.0891 |
| 28.1 | 0.4907 | 0.5029 | 0.0951 | 0.5713 | 0.6566 | 0.0829 |
| 29.1 | 0.491 | 0.5034 | 0.0918 | 0.5595 | 0.6128 | 0.0773 |
| 30.1 | 0.4912 | 0.504 | 0.0888 | 0.5484 | 0.5768 | 0.0722 |
| 31.1 | 0.4914 | 0.5045 | 0.086 | 0.5378 | 0.5462 | 0.0676 |
| 32.1 | 0.4916 | 0.505 | 0.0833 | 0.5278 | 0.5192 | 0.0635 |
| 33.1 | 0.4918 | 0.5054 | 0.0808 | 0.5182 | 0.4951 | 0.0597 |
| 34.1 | 0.492 | 0.5059 | 0.0784 | 0.5091 | 0.4732 | 0.0563 |
| 35.1 | 0.4922 | 0.5064 | 0.0762 | 0.5004 | 0.4531 | 0.0531 |
| 36.1 | 0.4923 | 0.5069 | 0.0741 | 0.4921 | 0.4345 | 0.0502 |
| 37.1 | 0.4925 | 0.5073 | 0.0721 | 0.4841 | 0.4172 | 0.0475 |
| 38.1 | 0.4926 | 0.5078 | 0.0702 | 0.4765 | 0.4011 | 0.0451 |
| 39.1 | 1.1044 | 0.5082 | 0.0684 | 0.4692 | 0.386 | 0.0428 |
| 40.1 | 1.0913 | 0.5086 | 0.0667 | 0.4621 | 0.3719 | 0.0407 |
| 41.1 | 1.0788 | 0.5091 | 0.0651 | 0.4554 | 0.3586 | 0.0387 |
| 42.1 | 1.0666 | 0.5095 | 0.0635 | 0.4489 | 0.346 | 0.0369 |
| 43.1 | 1.0548 | 0.5099 | 0.062 | 0.4427 | 0.3342 | 0.0352 |
| 44.1 | 1.0434 | 0.5103 | 0.0606 | 0.4366 | 0.323 | 0.0336 |
| 45.1 | 1.0324 | 0.5107 | 0.0593 | 0.4308 | 0.3124 | 0.0321 |
| 46.1 | 1.0217 | 0.5112 | 0.058 | 0.4252 | 0.3024 | 0.0308 |
| 47.1 | 1.0113 | 0.5116 | 0.0568 | 0.4198 | 0.2929 | 0.0295 |
| 48.1 | 1.0012 | 0.512 | 0.0556 | 0.4146 | 0.2838 | 0.0283 |
| 49.1 | 0.9915 | 0.5124 | 0.0545 | 0.4095 | 0.2752 | 0.0271 |
| 50.1 | 0.9819 | 0.5128 | 0.0534 | 0.4046 | 0.267 | 0.026 |
| 51.1 | 0.9727 | 0.5132 | 0.0523 | 0.3998 | 0.2592 | 0.025 |
| 52.1 | 0.9637 | 0.5136 | 0.0513 | 0.3952 | 0.2518 | 0.0241 |
| 53.1 | 0.955 | 0.514 | 0.0504 | 0.3908 | 0.2447 | 0.0232 |
| 54.1 | 0.9465 | 0.5144 | 0.0494 | 0.3864 | 0.238 | 0.0223 |
| 55.1 | 0.9382 | 0.5148 | 0.0485 | 0.3822 | 0.2315 | 0.0215 |
| 56.1 | 0.9301 | 0.5152 | 0.0477 | 0.3781 | 0.2253 | 0.0208 |
| 57.1 | 0.9222 | 0.5156 | 0.0468 | 0.3742 | 0.2194 | 0.02 |
| 58.1 | 0.9145 | 0.516 | 0.046 | 0.3703 | 0.2137 | 0.0194 |
| 59.1 | 0.907 | 0.5164 | 0.0453 | 0.3665 | 0.2083 | 0.0187 |

**Table 2**. The value of the attenuation coefficient with respect to frequency compared to various thermoelasticity theories

| **Frequency $\omega$** | $Q^{-1}$ **(C-T)** | $Q^{-1}$ **(L-S)** | $Q^{-1}$ **(G-L)** | $Q^{-1}$ **(G-N)** | $Q^{-1}$ **(DPL)** | $Q^{-1}$ **(TPL)** |
|---|---|---|---|---|---|---|
| 0.1 | -0.1429 | -0.1417 | -0.1407 | -0.0156 | -0.1405 | -0.0149 |
| 1.1 | -0.0539 | -0.0517 | 0.1649 | 0.0255 | -0.0231 | 0.0645 |
| 2.1 | -0.0381 | -0.039 | 0.0145 | 0.0212 | 0.0281 | -0.0588 |
| 3.1 | -0.0395 | -0.0433 | 5.0642 | -0.0341 | -0.0484 | -0.0872 |
| 4.1 | -0.0421 | -0.0494 | 8.0095 | -0.0483 | -0.0881 | -0.1053 |
| 5.1 | -0.0439 | -0.055 | 11.1732 | -0.0545 | -0.1108 | -0.114 |
| 6.1 | -0.0449 | -0.0597 | 17.5495 | -0.0578 | -0.1263 | -0.115 |
| 7.1 | -0.0453 | -0.0636 | 13.8256 | -0.0599 | -0.1377 | -0.1103 |
| 8.1 | -0.0453 | -0.0668 | 15.7837 | -0.0612 | -0.1464 | 0.2941 |
| 9.1 | -0.045 | -0.0695 | 10.5055 | -0.0623 | -0.1531 | 0.3214 |
| 10.1 | -0.0446 | -0.0717 | 9.6341 | -0.0632 | -0.1583 | 0.3488 |
| 11.1 | -0.0441 | -0.0736 | 9.0037 | -0.064 | -0.1623 | 0.3678 |
| 12.1 | -0.0435 | -0.0752 | 8.5273 | -0.0648 | -0.1653 | 0.3691 |
| 13.1 | -0.0429 | -0.0766 | 8.155 | -0.0656 | -0.1675 | 0.3502 |
| 14.1 | -0.0423 | -0.0779 | 7.8562 | -0.0663 | -0.1689 | 0.3187 |
| 15.1 | -0.0418 | -0.0791 | 7.6113 | -0.0671 | -0.1696 | 0.2848 |
| 16.1 | -0.0412 | -0.0802 | 7.407 | -0.0678 | -0.1698 | 0.2546 |
| 17.1 | -0.0406 | -0.0812 | 7.234 | -0.0685 | -0.1694 | 0.2298 |
| 18.1 | -0.0401 | -0.0822 | 7.0858 | -0.0692 | -0.1685 | 0.2099 |
| 19.1 | -0.0396 | -0.0832 | 6.9573 | -2.4806 | -0.1671 | 0.1937 |
| 20.1 | -0.0391 | -0.0841 | 6.8449 | -2.47 | -0.1654 | 0.1803 |
| 21.1 | -0.0387 | -0.085 | 6.7457 | -2.4602 | -0.1633 | 0.1691 |
| 22.1 | -0.0382 | -0.0858 | 6.6576 | -2.451 | -0.1609 | 0.1595 |
| 23.1 | -0.0378 | -0.0867 | 6.5788 | -2.4423 | -0.1582 | 0.1511 |
| 24.1 | -0.0374 | -0.0875 | 6.508 | -2.4341 | -0.1553 | 0.1438 |
| 25.1 | -0.0371 | -0.0883 | 6.4439 | -2.4263 | -11.522 | 0.1372 |
| 26.1 | -0.0367 | -0.0892 | 6.3857 | -2.4188 | -72.149 | 0.1313 |
| 27.1 | -0.0364 | -0.09 | 6.3326 | -2.4116 | -258.48 | 0.126 |
| 28.1 | -0.0361 | -0.0908 | 6.2839 | -2.4046 | --205.6 | 0.1211 |
| 29.1 | -0.0357 | -0.0916 | 6.2392 | -2.3979 | -207.43 | 0.1166 |
| 30.1 | -0.0355 | -0.0924 | 6.1979 | -2.3914 | -195.45 | 0.1125 |
| 31.1 | -0.0352 | -0.0932 | 6.1597 | -2.3851 | -292.40 | 0.1087 |
| 32.1 | -0.0349 | -0.094 | 6.1243 | -2.3789 | -259.12 | 0.1051 |
| 33.1 | -0.0347 | -0.0948 | 6.0913 | -2.3729 | -84.051 | 0.1018 |
| 34.1 | -0.0344 | -0.0956 | 6.0606 | -2.3671 | -55.813 | 0.0987 |
| 35.1 | -0.0342 | -0.0964 | 6.0318 | -2.3614 | -20.767 | 0.0958 |
| 36.1 | -0.034 | -0.0972 | 6.0048 | -2.3558 | -27.437 | 0.0931 |
| 37.1 | -0.0338 | -0.098 | 5.9795 | -2.3503 | -25.124 | 0.0905 |
| 38.1 | -0.0336 | -0.0988 | 5.9557 | -2.3449 | -23.459 | 0.0881 |
| 39.1 | -0.7987 | -0.0995 | 5.9333 | -2.3396 | -22.232 | 0.0858 |
| 40.1 | -0.7979 | -0.1003 | 5.9121 | -2.3344 | -21.312 | 0.0836 |
| 41.1 | -0.7971 | -0.1011 | 5.8921 | -2.3293 | -20.617 | 0.0815 |
| 42.1 | -0.7963 | -0.1019 | 5.8732 | -2.3243 | -20.091 | 0.0796 |
| 43.1 | -0.7956 | -0.1027 | 5.8552 | -2.3194 | -19.694 | 0.0777 |
| 44.1 | -0.7949 | -0.1035 | 5.8381 | -2.3146 | -19.400 | 0.0759 |
| 45.1 | -0.7943 | -0.1043 | 5.8219 | -2.3098 | -19.187 | 0.0742 |
| 46.1 | -0.7937 | -0.105 | 5.8064 | -2.3051 | -19.040 | 0.0726 |
| 47.1 | -0.7931 | -0.1058 | 5.7917 | -2.3005 | -18.94 | 0.071 |
| 48.1 | -0.7926 | -0.1066 | 5.7777 | -2.2959 | -18.901 | 0.0695 |
| 49.1 | -0.7921 | -0.1074 | 5.7642 | -2.2914 | -18.892 | 0.0681 |
| 50.1 | -0.7916 | -0.1081 | 5.7514 | -2.287 | -18.915 | 0.0667 |
| 51.1 | -0.7911 | -0.1089 | 5.7391 | -2.2826 | -18.967 | 0.0654 |
| 52.1 | -0.7907 | -0.1097 | 5.7274 | -2.2784 | -19.042 | 0.0642 |
| 53.1 | -0.7903 | -0.1104 | 5.7161 | -2.2741 | -19.138 | 0.0629 |
| 54.1 | -0.7899 | -0.1112 | 5.7052 | -2.2699 | -19.253 | 0.0618 |
| 55.1 | -0.7895 | -0.112 | 5.6948 | -2.2658 | -19.383 | 0.0606 |
| 56.1 | -0.7891 | -0.1127 | 5.6848 | -2.2617 | -19.528 | 0.0595 |
| 57.1 | -0.7888 | -0.1135 | 5.6752 | -2.2577 | -19.685 | 0.0585 |
| 58.1 | -0.7884 | -0.1142 | 5.666 | -2.2538 | -19.853 | 0.0575 |
| 59.1 | -0.7881 | -0.115 | 5.657 | -2.2499 | -20.031 | 0.0565 |

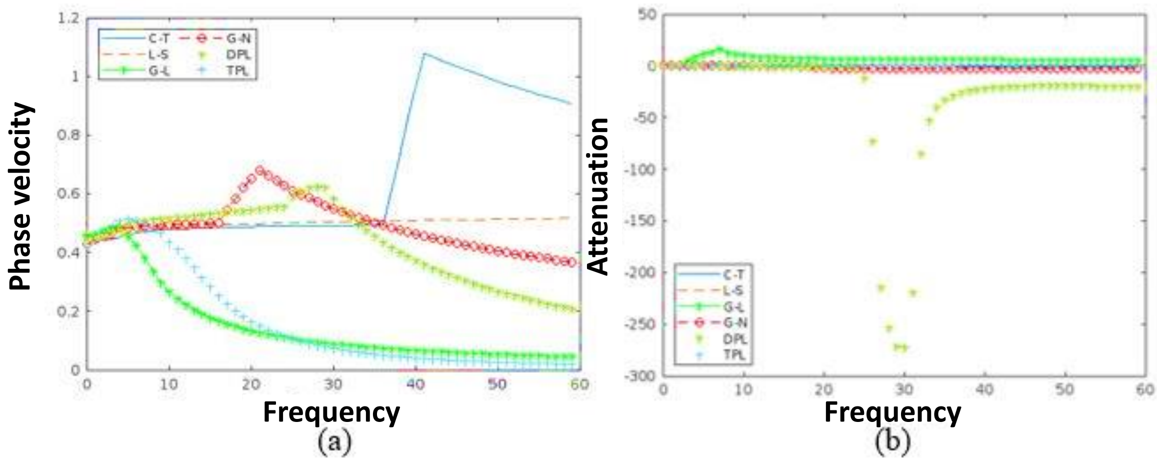


**Fig. 1.** Variation of phase velocities $V$ (m/s) and attenuations ($Q^{-1}$) with frequency $\omega$ (Hz) for different theories of thermoelasticity

From Fig. 1(a), the two-phase-lag theory, which increases smoothly and attains maximum value at $\omega = 27.1$, and after that phase, velocity starts deceasing and attains minimum value at $\omega = 59.1$. For the G-N theory, the value of $V$ increases smoothly in the range $0 \leq \omega \leq 15.1$and attains a sharp edge at $\omega = 19.1$. After this, it starts deceasing, constantly attaining the minimum value of phase velocity but higher than the two-phase-lag model velocity.

The L-S theory, depicts almost constant behaviour because there is very little change in phase velocity with an increase in values of frequency in the range $0 \leq \omega \leq 60$. The couple theory of thermoelasticity shows constant behaviour with very little difference in the value of phase velocity for frequency in the range $0 \leq \omega \leq 38.1$. After it, the constant increase in value $V$ in the range $38.1 \leq \omega \leq 48.1$ then decreases. Overall, phase velocity near 0.4 is attained by all theories of thermoelasticity $\omega = 0.1$, with the maximum value $V$ attained by coupled theory and the minimum value $V$ attained by three-phase-lag theory $\omega = 30.1$. The values of $V$ are very close to each other for the coupled L-S, G-N, and two-phase lag theories of thermoelasticity.

It depicts that in Fig. 1(b), the value of attenuation is near zero and attains negative values in the coupled and L-S theories. In the case of the three-phase lag theory, the value of attenuation attains a positive value near zero. In the case of the G-L theory, the value $Q^{-1}$ increases in the range $0.1 \leq \omega \leq 8.1$ and then starts to decrease with an increase in the value of frequency. In the case of the G-N theory, the value of attenuation decreases as the value of frequency increases.

Two-phase-lag theory attains a minimum value of $Q^{-1}$ at $\omega = 31.1$. This is the lowest value attained in comparison to other theories. In this case, a large fluctuation is observed. The maximum value of attenuation is attained $\omega = 8.1$ by the G-L theory of thermoelasticity. Two-phase theory attains a minimum value of attenuation coefficient with respect to other theories. The majority of values attained by all the theories are near to zero, either positive or negative, within the range $0.1 \leq \omega \leq 59.1$.

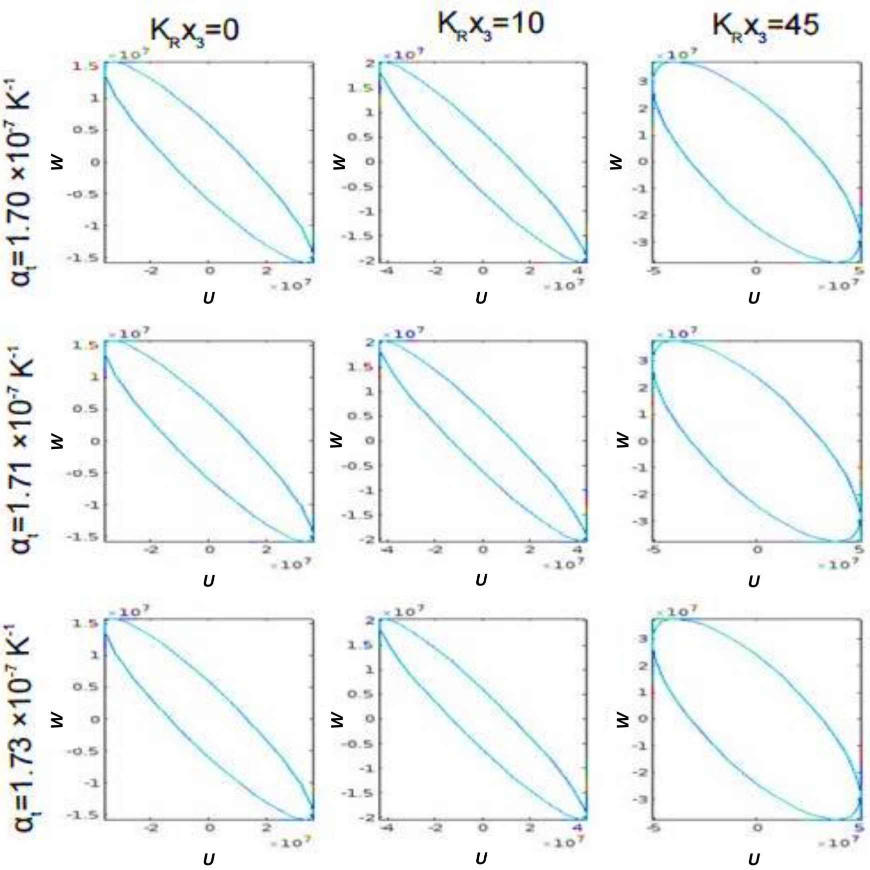


**Fig. 2.** Particle motion variation $(U, W)$ with depth $K_R x_3 = 0,\ 15,\ 45$ for couple theory of thermoelasticity

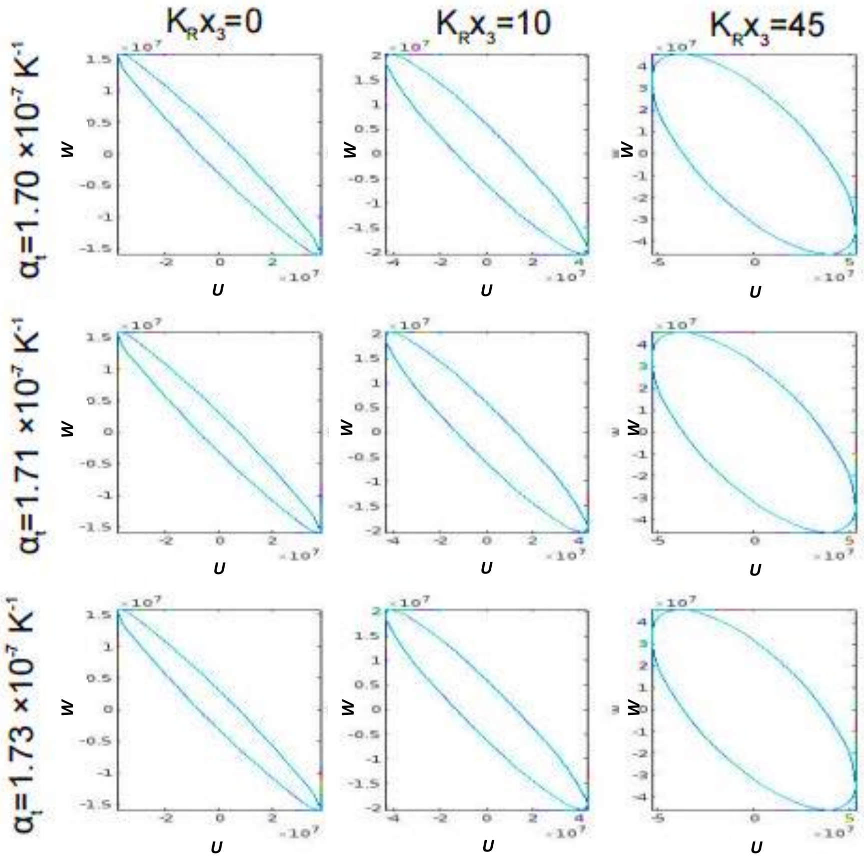


**Fig. 3.** Particle motion variation $(U, W)$ with depth $K_R x_3 = 0,\ 15,\ 45$ for L-S theory of thermoelasticity

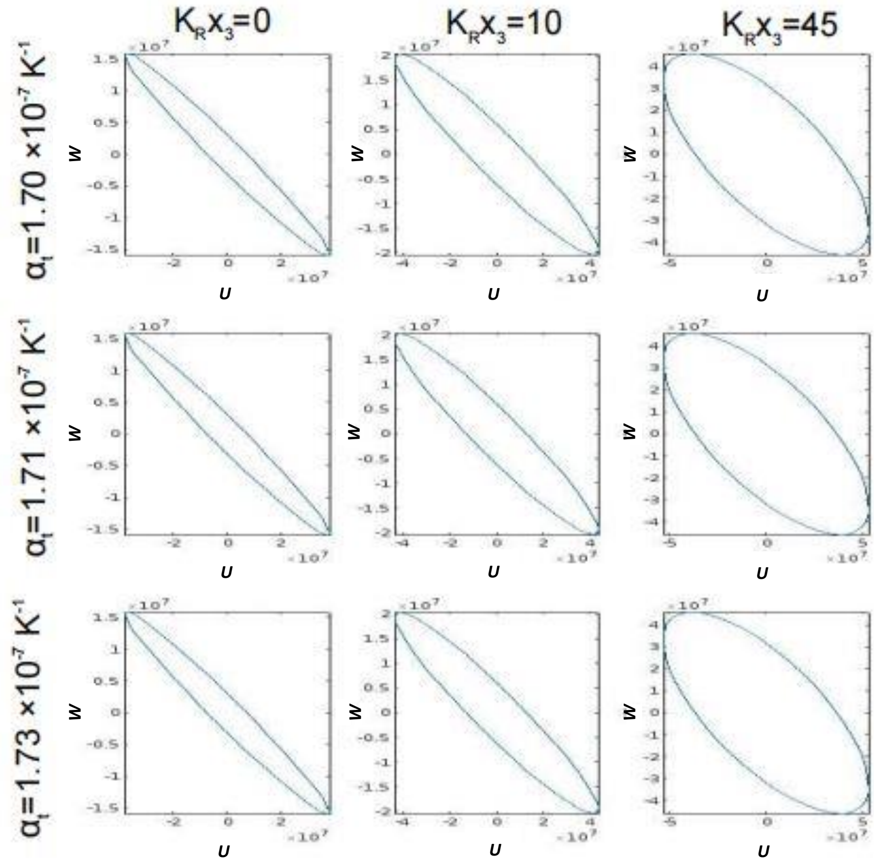


**Fig. 4.** Particle motion variation $(U, W)$ with depth $K_R x_3 = 0,\ 15,\ 45$ for G-L theory of thermoelasticity

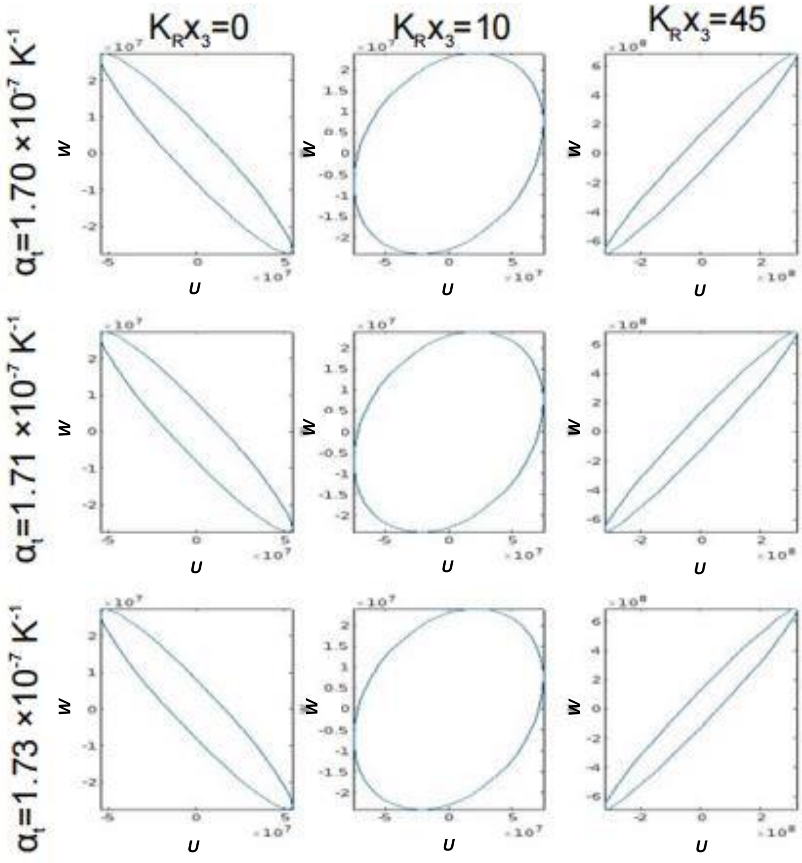


**Fig. 5.** Particle motion variation $(U, W)$ with depth $K_R x_3 = 0,\ 15,\ 45$ for G-N theory of thermoelasticity

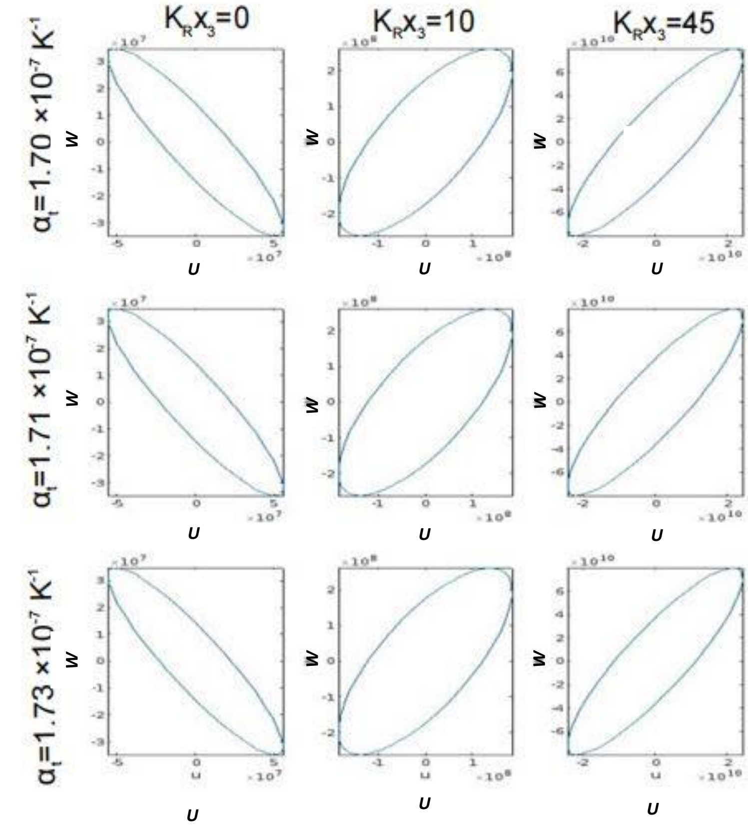

**Fig. 6.** Particle motion variation $(U, W)$ with depth $K_R x_3 = 0,\ 15,\ 45$ for two-phase -lag theory of thermoelasticity

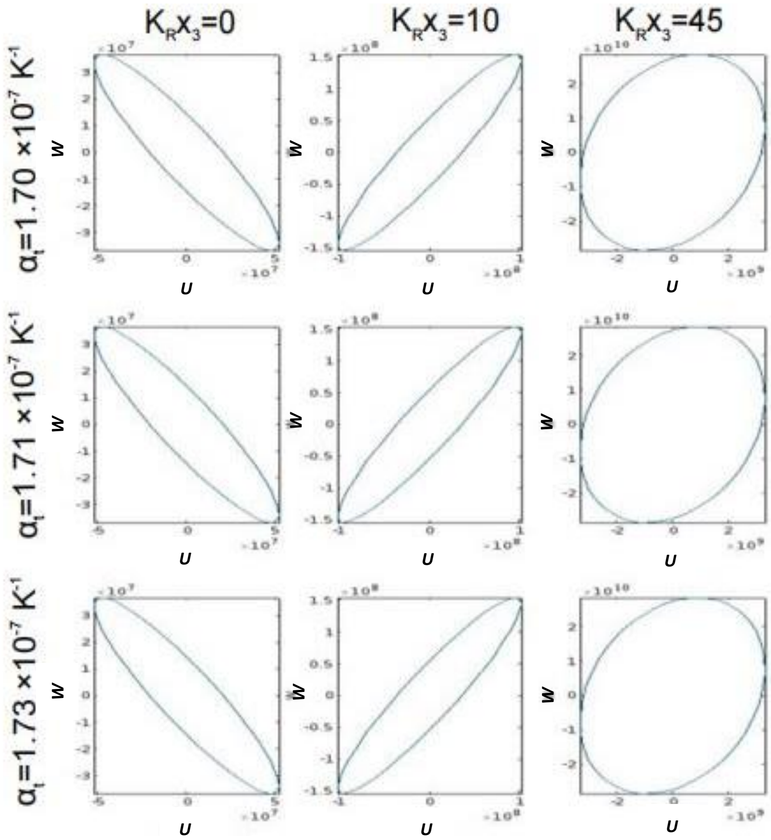

**Fig. 7.** Particle motion variation $(U, W)$ with depth $K_R x_3 = 0,\ 15,\ 45$ for three phase lag theory of thermoelasticity

## Path of particle

All the special cases discussed in the form of different theories of thermoelasticity are presented graphically by using the output of MATLAB software in Figs. 2–7. The polarisation of particle motion in two-dimensional space is represented by $(U, W)$.

Physical data of copper material used with frequency $\omega = 2\pi$ and parameter $\Phi$ varies in $0.2\,\pi$. Different depths vary as follows: $K_R x_3 = 0,\ 15,\ 45$. The coefficient of thermal linear expansion varies as follows: $\alpha_t = (1.70 \times 10^{-7},\ 1.71 \times 10^{-7},\ 1.73 \times 10^{-7})\ \mathrm{K}^{-1}$. For one fixed value of depth, we present three figures for three different Coefficient of thermal linear expansion in such a way 9 figures obtained for three different depths in Figs. 2–7. Computed for coupled theory, L-S theory, G-L theory, G-N theory, two-phase-lag model, and three-phase-lag model of thermoelasticity.

In Fig. 2, it is observed that in the coupled theory of thermoelasticity, the motion of particles for the polarisation of the Rayleigh wave traces an elliptic path and tilt between the axes. Firstly, it shrinks and then expands into an elliptic shape with an increase in depth. In Fig. 3, it is depicted that in the case of the L-S theory of thermoelasticity, the motion of particles attains almost the same shape and size for two different values of depth and finally expands as they go deeper. As coupled theory is extended by L-S and G-L to generalise the theory of thermoelasticity, there is very little difference between these theories theoretically and numerically for the considered depth and coefficient of thermal linear expansion.

From Fig. 4, it is depicted that, using the G-L theory, the path of particle motion is tilted with an elliptic shape and becomes broad as we go deeper into the medium. In Fig. 5, elliptic path is covered by the particle during process of polarisation with tilt in opposite direction at depth $K_R x_3 = 0$ and $K_R x_3 = 45$.

In Fig. 6, the effect of the two-phase-lag theory at the depth $K_R x_3 = 0$ of the elliptic path is obtained between the axes. But as you go deeper, the shape obtained is opposite to the earlier and broad, then finally shrinks at the depth $K_R x_3 = 45$. In Fig. 7, it is observed that in the case of the three-phase-lag theory, the path of the particle is tilted between the axes. As they start going deep, the path of the particle remains the same but opposite in direction, and at the last motion, the particle is such that they obtain a broad elliptic shape.

## Conclusion

The present work examines the coupled, L-S, G-L, G-N, two-phase-lag, and three-phase-lag theories of thermoelasticity under Rayleigh wave propagation in an isotropic homogenous medium using the compact form of the heat conduction equation. The dispersion equation for isothermal surfaces has irrational terms in it. We rationalise this equation to get a polynomial equation. Roots that meet both equations are filtered by examining the decay qualities of those roots in depth. The dispersive nature and inhomogeneous nature of Rayleigh waves are verified. Comparing different theories is important while researching the Rayleigh wave's distinctive characteristics.

The isothermal surface increases the speed of the Ryleigh wave. To compare various thermoelasticity theories, the phase velocity, attenuation coefficients, and particle motion path are computed numerically and graphically shown. It is observed that the phase velocity attains its maximum value in the case of coupled theory and its minimum value in the case of the three-phase lag theory. L-S theory shows constant behaviour as frequency increases. For couples, L-S, G-N, and two-phase-lag theories, the values of attenuation coefficient are very close, due to which curves are overlapping. The maximum value is attained by the G-L model, and the minimum value is attained by the two-phase-lag model. The effect of different theories is significant for the particle motion of the Rayleigh wave and shows an elliptic path for different depths and the coefficient of thermal linear expansion. In the G-L theory of thermoelasticity, amplitude increases as we go deeper into the medium. One of the surface waves that is highly useful for researching numerous aspects of an earthquake is the Rayleigh wave.

## CRediT authorship contribution statement

**Vandana Gupta**: conceptualization, investigation, supervision, data curation; **Manoj Kumar**: investigation, supervision, data curation; **Shruti Goel:** writing – review & editing, writing – original draft.

## Conflict of interest

The authors declare that they have no conflict of interest.

## References

1. Biot MA. Thermoelasticity and irreversible thermodynamics. *Journal of Applied Physics*. 1956;27(3): 240–253.
2. Green AE, Lindsay KA. Thermoelasticity. *Journal of Elasticity*. 1972;2: 1–7.
3. Lord H, Shulman Y. A generalised dynamical theory of thermoelasticity. *Journal of Mechanical Physics and Solids*. 1967;15(5): 299–309.
4. Green AE, Naghdi PM. Thermoelasticity without energy dissipation. *Journal of Elasticity*. 1993;31: 189–208.
5. Hetnarski RB, Ignaczak J. Generalized thermoelasticity. *Journal of Thermal Stresses*. 1999;22(4-5): 451–476.
6. Ignaczak J, Ostoja-Starzewski M. *Thermoelasticity with Finite Wave Speeds*. Oxford University Press; 2009.
7. Tzou DY. A unified approach for heat conduction from micro to macro-scales. *Journal of Heat Transfer*. 1995;117(a): 8–16.
8. Choudhuri SKR. On thermoelastic three phase lag model. *Journal of Thermal Stresses*. 2007;30(3): 231–238.
9. Deresiewicz H. Effect of boundaries on waves in a thermo-elastic solid: Reflection of plane waves from plane boundary. *Journal of Mechanical Physics and Solids*. 1960;8(3): 164–172.
10. Sinha AN, Sinha SB. Reflection of thermoelastic waves at a solid half space with thermal relaxation. *Journal of Physics of the Earth*. 1974;22(2): 237–244.
11. Sinha SB, Elsibai KA. Reflection of thermoelastic waves at a solid half-space with two thermal relaxation times. *Journal of Thermal Stresses*. 1996;19(8): 763–777.
12. Sharma JN, Kumar V, Chand D. Reflection of generalized thermoelastic waves from the boundary of a half-space. *Journal of Thermal Stresses*. 2003;26(10): 925–942.
13. Singh MC, Chakraborty N. Reflection of a plane magneto-thermoelastic wave at the boundary of a solid half-space in presence of initial stress. *Applied Mathematics and Modelling*. 2015;39(5-6): 1409–1421.
14. Wei W, Zheng R, Liu G, Tao H. Reflection and refraction of P wave at the interface between thermoelastic and porous thermoelastic medium. *Transport in Porous Media*. 2016;113: 1–27.
15. Li Y, Li L, Wei P, Wang C. Reflection and refraction of thermoelastic waves at an interface of two couple-stress solids based on Lord-Shulman thermoelastic theory. *Applied Mathematics and Modelling*. 2018;55: 536–550.

16. Rayleigh L. On waves propagated along the plane surface of an elastic solid. *Proceeding of the Royal Society. London, Ser. A.* 1885;17: 4–11.
17. Sinha AN, Sinha SB. Velocity of Rayleigh waves with thermal relaxation in time. *Acta Mechanica.* 1975;23: 159-166.
18. Sharma MD. Phase velocity and attenuation of plane waves in dissipative elastic media: solving complex transcendental equation using functional iteration method. *International Journal of Engineering Science and Technology.* 2010;3(2): 130–136.
19. Tomita S, Shindo Y. Rayleigh waves in magneto-thermoelastic solids with thermal relaxation. *International Journal of Engineering Science and Technology.* 1979;17(2): 227–232.
20. Chadwick P, Windle DW. Propagation of Rayleigh waves along isothermal and insulated boundaries. *Proceeding of the Royal Society London.* 1964;280(1380): 47–71.
21. Dawn NC, Chakraborty SK. On Rayleigh wave in Green-Lindsay's model of generalized thermoelastic media. *Indian Journal of Pure and Applied. Mathematics.* 1988;20(3): 273–286.
22. Ahmed SM. Rayleigh waves in a thermoelastic granular medium under initial stress. *International Journal of Mathematical Science.* 2000;23: 627–637.
23. Sharma JN, Walia V, Gupta SK. Effect of rotation and thermal relaxation on Rayleigh waves in piezothermoelastic half space. *International Journal of Mechanical Science.* 2008;50(3): 433–444.
24. Abd-alla AN, Al-dawy AAS. Thermal relaxtion times effect on Rayleigh waves in generalised thermoelastic media. *Journal of Thermal Stress.* 2001;24(4): 367–382.
25. Mahmoud SR. Influence of rotation and generalized magneto-thermoelastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field. *Meccanica.* 2012;47: 1561–1579.
26. Noguchi S, Maeda T, Furumura T. Ocean-influenced Rayleigh waves from outer-rise earthquakes and their effects on durations of long-period ground motion. *International Journal of Geophysical.* 2016;205(2): 1099–1107.
27. Bucur AV, Passarella F, Tibullo V. Rayleigh surface waves in the theory of thermoelastic materials with voids. *Meccanica.* 2014;49: 2069–2078.
28. Zhao W, Chen W, Yang D, Gao H, Xie P. Analytical solution for seismic response of tunnels with composite linings in elastic ground subjected to Rayleigh waves. *Soil Dynamics and Earthquake Engineering.* 2022;153: 107113.
29. Li X, Li Q, Shen H, Yang F, Zhang X. Research on oil exploration seismic Rayleigh wave imaging based on multi-channel analysis of surface waves and genetic-damped least squares joint inversion. *Journal of Applied Geophysics.* 2022;202: 104670.
30. Singh B, Verma S. On Propagation of Rayleigh Type Surface Wave in Five Different Theories of thermoelasticity. *International Journal of Applied Mechanics and Engineering.* 2019; 24(3): 661–673.
31. Sharma S, Kumari S, Singh M. Rayleigh wave propagation with the effect of initial stress, magnetic field and two temperatures in the dual phase lag thermoelasticity. *Journal of Advances in Mathematics.* 2020;9(9): 7535–7545.
32. Kumar R, Gupta V. Rayleigh waves in generalized thermoelastic medium with mass diffusion. *Canadian Journal of Physics.* 2015;93(10): 1039–1049.
33. Kumar R, Singh K, Singh D. Propagation of Rayleigh waves in a micropolar thermoelastic half-space with impedance boundary conditions. *Materials Physics and Mechanics*. 2018;35(1): 115–125.
34. Haque I, Biswas S. Rayleigh waves in nonlocal porous thermoelastic layer with Green- Lindsay model. *Steel and Composite Structures.* 2024;50(2): 123–133.
35. Saeed T, Khan MA, Alzahrani ARR, Jahangir A. Rayleigh wave through half space semiconductor solid with temperature dependent properties. *Physica Scripta.* 2024;99: 025208.
36. Singh K, Kashyap M. Memory effect on Rayleigh waves propagation in a micro polar thermoelastic half space. *Mechanics of Solids.* 2023;58: 1228–1238.
37. Kumar R, Gupta V. Uniqueness, reciprocity theorems and plane wave propagation in different theories of thermoelasticity. *Journal of Applied Mechanics and Engineering.* 2013;18(4): 1067–1086.
38. Sharma MD. Propagation and attenuation of Rayleigh waves in generalized thermoelastic media. *Journal of Seismology.* 2014;18: 61–79.
39. Ewing WM, Jardetzky WS, Press F. Elastic layers in Layered Media. New York: McGraw-Hill Company; 1957.