Positioning of a sphere in a rotating cylinder under condition of vibrational hydrodynamic top

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Abstract

The dynamics of a light centrifuged sphere in a rotating inclined cylinder filled with liquid under transversal vibrations is experimentally investigated. The sphere and the cavity rotate at different angular velocities, and in the absence of vibration the cavity velocity is always greater than the sphereвЂ™s one. This phenomenon was called the vibrational hydrodynamic top [1].

The intensification of the lagging motion of a sphere and the excitation of the outstripping differential rotation are possible under transversal vibrations. It occurs in the resonant areas where the frequency of vibrations equals to the fundamental frequency of the system. The resonant areas are determined by the ratio of the frequency of vibrations to the angular velocity of the cavity rotation $n \equiv \Omega_{vib}/\Omega_{rot}$. The position of the centrifuged sphere in the center of the cylinder is unstable [2]. In the threshold of outstripping motion excitation the sphere is shifted from the center to the one of the end-walls and is settled at definite distance from it. The position of the sphere is determined by the dimensionless frequency of vibrations n and depends on the intensity and structure of the averaged fluid flow.

In inclined rotating cylinder the axial component of the gravity force appears however a light sphere does not float to the end-wall but saves the quasi-stationary position at a definite distance from it. It makes possible to create a vibration suspension of a light sphere in a vertical rotating cavity under transversal vibrations. It is found that in the wide range of angles of the cavity inclination the sphere position is determined by dimensionless rotation rate.

1 Introduction

Rotating hydrodynamic systems cause a great interest because of their wide distribution in nature. A wide range of natural frequencies provides the possibility to control such systems using the vibrations [3]. For instance the action of transverse vibrations on the free boundary of the centrifuged liquid [4] leads to the excitation of an azimuthal wave. A similar resonance effects occur when the free flowing medium [5] or a light cylindrical body [1] is in rotating system instead of a gas phase. In the last case the vibration leads to the oscillation of the body and the emergence of its intensive differential rotation relative to the cavity, called the vibrational hydrodynamic top. Description of the differential rotation in the two-dimensional formulation is given in [1]. The presence of a rotating force field in P° rotating frame leads to the circular oscillations of the body. The arising inertial azimuthal wave in the fluid causes the pulsating motion in the viscous boundary layer. This leads to the excitation of an averaged torque, spinning up the body. The direction of body rotation (outstripping or lagging) is determined by the direction of the azimuthal wave. The existence of the areas of the lagging and outstripping rotation is natural for all centrifuged systems inhomogeneous in density. The resonance areas are defined by the ratio of vibration frequency to the rotation speed $n \equiv \Omega_{vib}/\Omega_{rot}$. The outstripping rotation of the body with respect to the cavity is excited if $n > 1$, the lagging one – $n < 1$.

If the top has a spherical shape [2] the vibrations results in excitation of its differential rotation and positioning on the rotational axis. The positioning of bodies is an actual problem, particularly under the microgravity conditions. The phenomenon of "acoustic levitation" of liquid droplets in the gravity field [6, 7] could be an example. The levitation is carried out by the acoustic pressure in the standing wave. In the microsystem an oscillating electric field in the quadrupole traps is used for the positioning of electrically charged particles [8].

The rotation of the cavity qualitative changes the law of motion and wrap of the bodies. In the rotating fluid the emersion speed of a light body is much smaller than in a non-rotating one [9, 10]. The additional drag force due to the formation of so-called Taylor–Proudman column appears [11].

In this work we study the behavior of a light spherical body in an inclined rotating cylinder filled with a fluid and subjected to vibration perpendicular to the rotation axis. The sphere has the differential rotation with respect to the cavity due to the vibration. The differential rotation also generates the Taylor–Proudman column. Unlike the classical column with almost solid-body rotation, the column in the problem of the vibrational top has a complex vortex internal structure [2]. We found that under these conditions the body can take the suspended state even in the vertical cavity.

2 Experimental setup and techniques

In a cylindrical cavity of circular cross section *1* (fig. 1) the light spherical body *2* is placed. The cavity filled with a liquid is set in a horizontal position on the table *3* of an electrodynamic vibrator, providing the translational vibrations perpendicular to the rotation axis. The length and the radius of the cavity are $L = 72.0$ mm and $R = 26.0$ mm respectively, the radius of the sphere is $r = 17.7$ mm, its average density is $\rho_s = 0.17$ g/cm^3. The relative size of the sphere is $2r/L = 0.5$. The working fluid is water.

A stepper motor *4* lead the cavity to the uniform rotation with frequency $f_{rot} = \Omega_{rot}/2\pi$. The instability of the cavity rotation is less than 0.001 rps. The motor rotation speed is controlled by a generator. The cavity rotation speed is such that the sphere is centrifuged and rotates without touch of the side walls. The sphere angular velocity $f_s = \Omega_s/2\pi$ is different from the cavity rotation speed. The stroboscopic light lamp *5* is used for illumination. The frequency of vibrations is f_{vib}. The amplitude of vibration b_{vib} is calculated using the signal of accelerometer *6*.

In the experiment the body position $x = (x_2 - x_1)/(x_2 + x_1)$ with respect to the cavity end walls (x_1, x_2 – the distance between the left and the right end walls of the cavity to the closest poles of the sphere) and the relative angular velocity of the sphere $\Delta f = f_s - f_{rot}$ are measured. All experiments were conducted in the resonance areas where the frequency of vibrations coincides with one of the natural frequencies of the sphere and the differential rotation Δf (outstripping or lagging) is large [2]. For the chosen problem the resonant frequencies of outstripping and lagging rotation are $n = 2.0$ and $n = 0.8$.

The experiment was conducted as follows. In one case the angle of the cavity inclination α is changing continuously at constant frequency of rotation and frequency and amplitude of vibrations. The experiment is repeated for different values of f_{vib} and b_{vib}. In another case at fixed frequency of vibration and angle O± the amplitude of vibration b_{vib} is varied.

For the outstripping rotation the cavity speed and the vibration frequency are

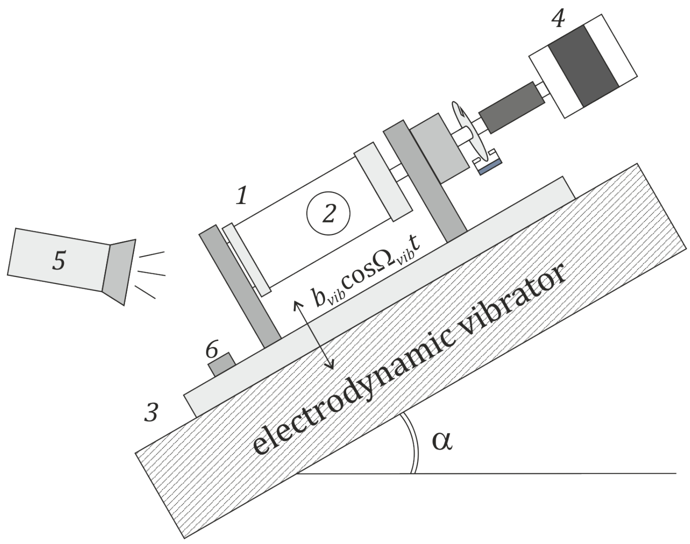

Figure 1: The scheme of the experimental setup

$f_{rot} = 17.5$ rps and $f_{vib} = 35$ Hz. For the lagging rotation $f_{rot} = 31.0$ rps and $f_{vib} = 25$ Hz. The amplitude of the vibrations varies in the range $b_{vib} = 0.01 - 0.80$ mm.

3 Experimental results

The rotation speed of the sphere in the laboratory frame is always less then the cavity speed ($\Delta f < 0$) in case of rotation about a horizontal axis at the absence of vibrations. The solid takes the quasistationary position at some distance from the cavity end walls. The position depends on the dimensionless frequency $\omega = \Omega_{rot} r^2/\nu$ where ν is the kinematic viscosity. The sphere dynamics under vibration depends on the parameter n. The area of intense outstripping rotation ($\Delta f > 0$) is characterized by a significant shift of the sphere out the cavity center (the horizontal case) [2]. The displacement can occur to any end. In the lagging resonant rotation area the sphere position is at almost equal distance from the ends, $|x| < 0.2$. Outside the resonance areas the sphere position coincides with the non-vibration case.

For the small amplitudes of vibrations the sphere retains practically the same quasistationary position (fig. 2a) with the cavity deviation from the horizon. With increasing the cavity inclination the situation remains unchanged until a certain critical angle α^*, at which the sphere in the threshold manner moves along the axis to the end wall, "floats" (fig. 3a). With decrease of the inclination angle the separation from the upper wall occurs with hysteresis at much lower value of α (fig. 3a, black symbols). The new quasistationary position of the sphere is much closer to the upper end wall. The transition to the initial state x takes place only after the return of the cavity in the horizontal position. The sphere differential velocity changes simultaneously with the position (fig. 3b). Floating and separation of the sphere from the top wall is accompanied by the threshold change in the Δf. In the range of inclination angle $\alpha = 20$–40^0 the behavior of the sphere is unsteady. The rotation speed Δf (at $\alpha = const$) is changing in time while the x is constant.

The increase of the vibration amplitude leads to the grow of the differential rotation speed and the critical angle α^*. Under the intensive vibration the sphere could remain in the quasiequilibrium position at some distance from the upper end wall of the cavity even in the vertical cylinder (fig. 3c). Floating occurs only at low vibration amplitude b_{vib}

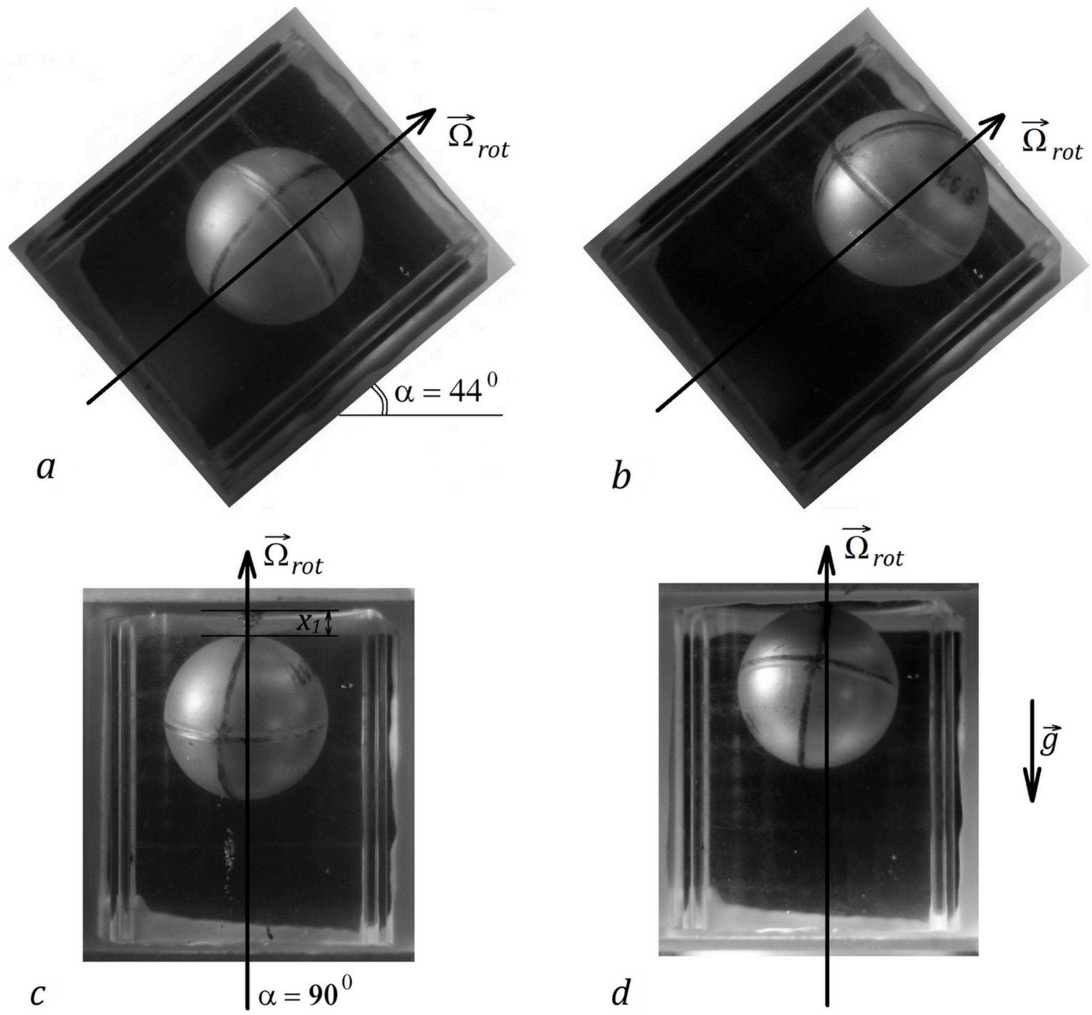

Figure 2: Photographs of the sphere quasistationary position at n = 0.8; (a), 0.29 (b), 0.70 (c) and 0.63 mm (d)

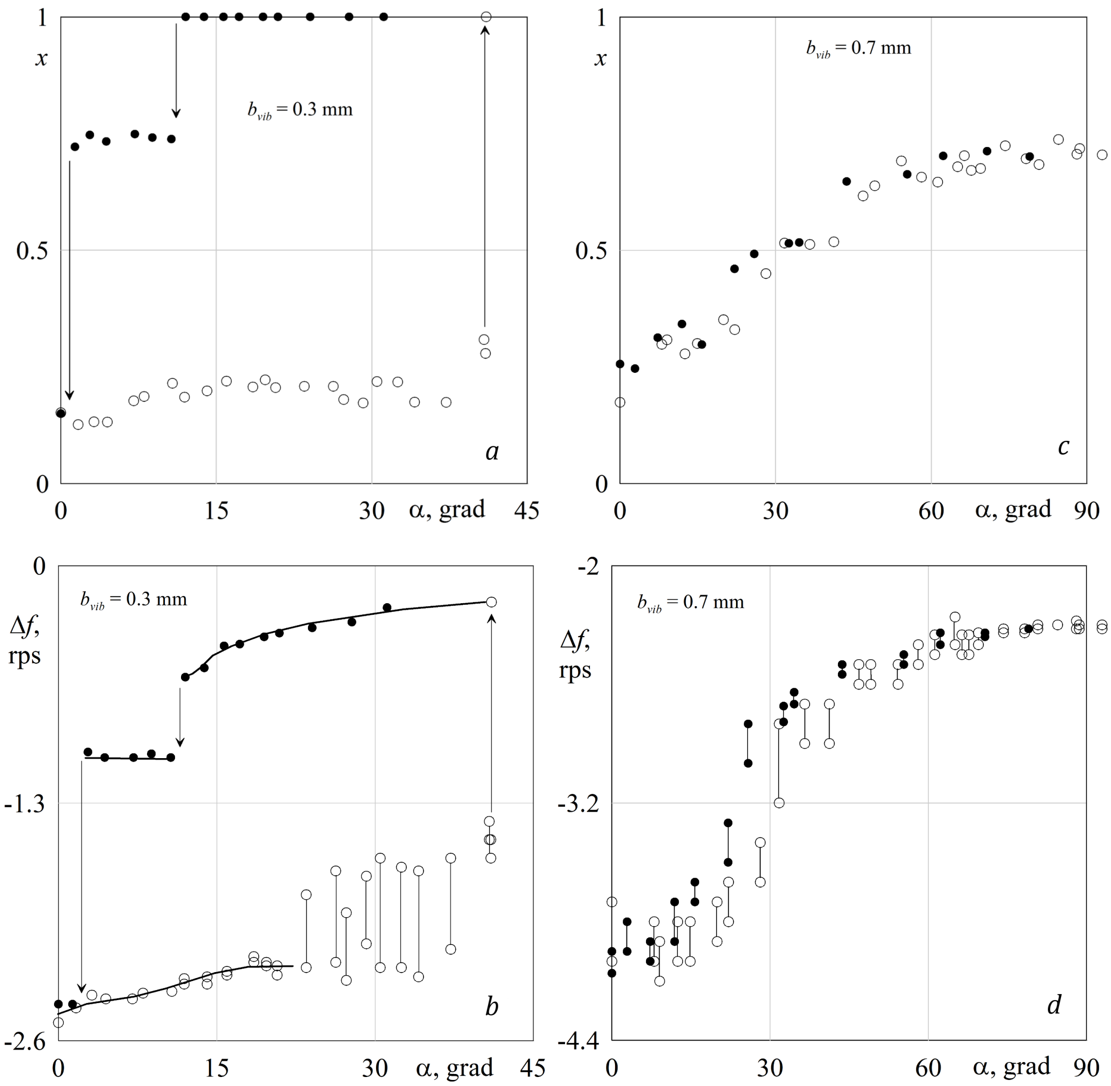

Figure 3: The position of the sphere relative to the cavity ends (a, c) and the differential rotation speed (b, d) as a function of the angle α for different vibration amplitudes at $n = 0.8$; dark and empty symbols correspond to the results for increasing and decreasing α; area of sphere unsteady rotation is indicated by lines connecting the minimum and maximum values Δf; the transitions from one quasisteady state to another are shown by arrows

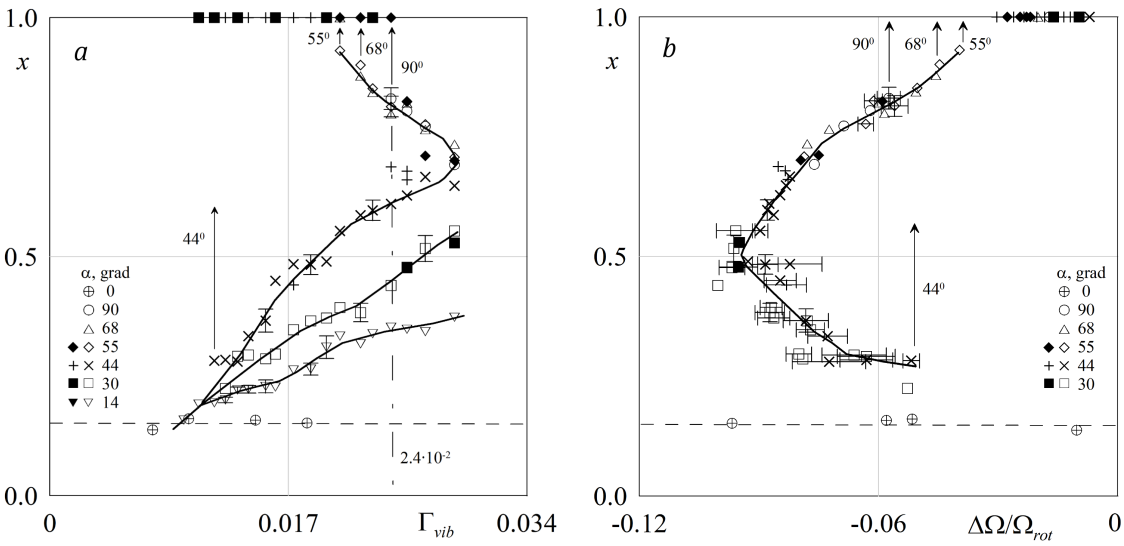

Figure 4: The position of the body versus the dimensionless vibration acceleration (a) and dimensionless sphere speed (b) at different α ($n = 0.8$); dark and light symbols correspond to increase and decrease of b_{vib}; solid arrows indicate the thresholds of the sphere float to the upper end wall

(fig. 2d). Thus, the vibration allows the suspension the light sphere in the rotating vertical cylindrical cavity with fluid.

4 Discussion of results

The intensity of the vibration is determined by the dimensionless parameter $\Gamma_{vib} = (b_{vib}\Omega_{vib}^2)/(r\Omega_{rot}^2)$ that characterizes the ratio of the vibrational acceleration to the centrifugal one. The increase of Γ_{vib} leads to the growth of dimensionless velocity of the differential rotation $|\Delta\Omega/\Omega_{rot}|$. In the inclined cavity it also leads to a change in the sphere position x (fig. 4a). The position x in the horizontal cavity at $n = 0.8$ practically does not depend on Γ_{vib} (fig. 4a).

The coordinate x changes with Γ_{vib} nonmonotonically. For small α ($< 45^0$) with Γ_{vib} growth the sphere shifts to the upper end of the cavity. The greater is cavity inclination angle at definite Γ_{vib}, the larger is x. For large α ($> 45^0$) the dependence is opposite, the coordinate x decreases with Γ_{vib}. All points for different values α fall down on a single curve. In the vertical cavity ($\alpha = 90^0$) the vibrational suspension of the sphere is possible at $\Gamma_{vib} > 0.024$.

The dependence of the body position x on the dimensionless speed $\Delta\Omega/\Omega_{rot}$ is shown in fig. 4b. The axial component of the lift force acting on the sphere $F_n = (\rho_L - \rho_s)\, V_s g \sin\alpha$ grows with α. However the experimental points for different α are in agreement and fall down on one curve. Thus, the position of the sphere is independent of the angle of inclination, and is completely determined by the dimensionless differential rotation speed. The body could get stable position at different distances from the end wall at the same values of $\Delta\Omega/\Omega_{rot}$.

5 Conclusion

The behavior of the light spherical body in the rotating inclined cylinder with fluid under vibration perpendicular to the axis of rotation is experimentally investigated. It is found that the vibration results in the intensive differential rotation of the sphere and quasistationary positioning at definite distance from the end walls. With increase of the cavity inclination angle the sphere "suspension" state remains up to some critical value α^*, which grows with the vibration intensity. At $\Gamma_{vib} > 0.024$ the suspension of the light sphere in the vertical cylindrical cavity is possible.

Acknowledgements

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