High velocity gliding of a plate with final length cavity formation

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Abstract

The two-dimensional problem of thin body motion in fluid parallel to the boundary at a distance comparable with the length of the body was regarded. The fluid was assumed occupying infinite semi-space, gravity was neglected as compared with fluid inertia. The solution was obtained for a problem of plate motion in compressible fluid at a definite depth under free surface, constant velocity and inclination angle. The formation of final length cavity behind the body was taken into account. Vapor pressure in the cavity is assumed to be less than the pressure on free surface. The solution allows determining drag and lift forces. The relation between the body length and the cavity length in the case of small depth or high compressibility was obtained. It was determined that the forces increase when the pressure in the cavity decreases. When the pressure in the cavity tends to the pressure on the free surface the length of the cavity infinitely increases and forces tend to the values obtained in independent solutions with infinite cavities. The increase of depth under free surface and decrease of compressibility brings to decrease of cavity length.

1 Introduction

The problem of gliding over the surface of water of infinite and finite depth was regarded within the frames of linear [1-4] and non-linear [5, 6] statements, and found its generalized classical solution in [7]. High speed streaming flows accounting for fluid compressibility were investigated in [8-10]. The problems regarded in [10] include both cases of positive and negative angles of attack and describe underwater motion of a plate. The solutions relied heavily on the fact that cavity behind the gliding body has infinite length. The problem of final length cavity formation near a plate moving in incompressible fluid was solved numerically in [11-13]. In the present paper the analytical solution allowing determine drag and lift forces of a plate moving in compressible fluid near free surface with final length cavity formation was obtained.

The current problem has many practical applications, such as determining resistance and lift forces being the function of the depth in underwater motion of a bullet or shell. The problem is relevant to surface or underwater high velocity gliding of thin wing, which is often used to reduce resistance of the glider.

2 Mathematical statement of the problem

The two-dimensional problem of thin body motion in the presence of free surface is regarded. It is assumed that the wing is moving with constant velocity $\underline{V}_0$ in an ideal compressible fluid near a free surface. In a motionless coordinate system adiabatic gas flow is described by the continuity and Euler equations. The angle $|\alpha^{\pm}| \ll 1$ and mass forces are considered to be negligibly small. These assumptions make the flow field to be potential. Boundary conditions should be satisfied on the free surface, on the body surface contacting fluid and in the cavity. On the free surface and in the cavity constant pressure is assumed, on the fluid-body contact streaming condition of the equality of normal velocity component. Behind the cavity fluid streams from both sides of the body merge thus creating a uniform flow without any slipping one fluid against the other.

The coordinate system and flow scheme are shown in Figure 1. In movable coordinate system $x = x' + V_0 t, y = y'$ connected with the wing the gas flow can be considered stable.

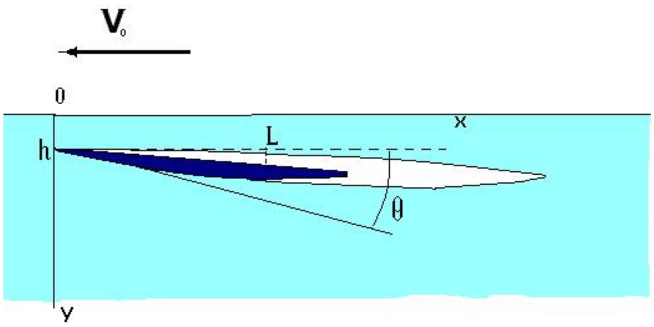

Figure 1: Thin body motion in fluid parallel to free surface with closed cavity formation.

Flow potential under the condition of steady-state flow satisfies the equation

$$V_0^2 \frac{\partial^2 \varphi}{\partial x} = a^2 \left(\frac{\partial^2 \varphi}{\partial x} + \frac{\partial^2 \varphi}{\partial y}\right), \tag{1}$$

and fluid pressure is determined

$$P - P_0 = \rho_0 V_0 \frac{\partial \varphi}{\partial x}. \tag{2}$$

Boundary conditions look as follows

$$\begin{gathered} y = 0, -\infty < x < \infty : \quad \frac{\partial \varphi}{\partial x} = 0; \\ y = h^+, 0 < x < L : \quad \frac{\partial \varphi}{\partial y} = V_0 \sin\theta; \\ y = h^-, 0 < x < S; \quad y = h^+, L < x < S : \frac{\partial \varphi}{\partial x} = -\frac{\Delta P}{\rho_0 V_0}; \\ y = h^{\pm}, L < x : \quad \frac{\partial \varphi}{\partial x} = 0, \end{gathered} \tag{3}$$

where V_0 is the mean streaming velocity, $\varphi(x, y, t)$ is the potential of the disturbed flow induced by the relative motion of the body, P_0, ρ_0 are pressure and density in quiescent fluid, $\Delta P = C_{cav.min} \frac{\rho_0 V_0^2}{2}$, where $C_{cav.min}$ is the minimal cavitation number, which is determined by the thermo-physical properties of liquid, amount of solute gases, presence of impurities, fluid temperature, etc.

Thus equation (1) with boundary conditions (3) present a closed form statement of the problem.

3 Problem solution

The solution will be developed in the form of a real part for the analytical function of a complex variable $\varphi(x,y) = Re\Phi(z), z = x+iy$. Actually, it is necessary to develop first derivative of the analytical function, which could be denoted as $T(z) = \Phi^{'}(z)$.

In case of the body moving near free surface the conformal mapping of the semi-plane $y > 0$ with a cut $y = \pi, x > 0$ on the upper semi-plane $Imw > 0$ (Figure 2), which could be performed with the function:

$$z = \pi i + w - \ln w - 1, \quad Imw > 0, \quad w = u + iv$$

can be used to obtain a new boundary problem in the new plane for $T(z(w))$ function. The development of the analytical function is reduced to Riemann - Hilbert problem, which solution is non-unique.

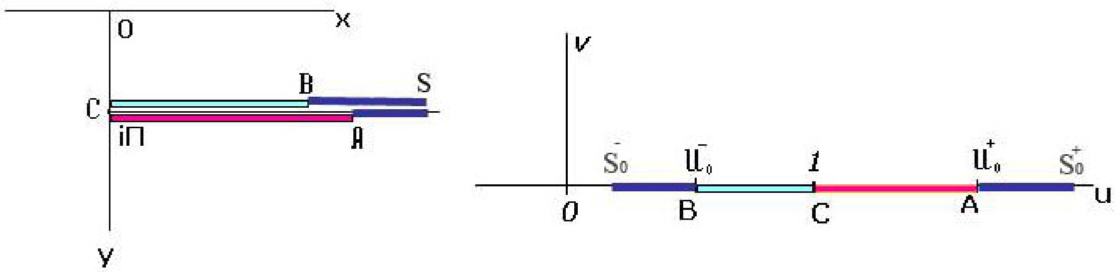

Figure 2: Conformal mapping of the semi-plane $y > 0$ with a cut $y = \pi, x > 0$ on the upper semi-plane.

The solution could be developed in the following form:

$$T(w) = \sqrt{\frac{u-1}{u_0^+ - 1}} Q(w),$$

where $Q(w)$ tends to zero at infinity. The solution is limited at infinity, and it is equal to zero at the front edge.

The solution for $ReQ(w)$ function is reduced to Dirichlet boundary problem, which has a unique solution.

Thus it is necessary to develop a harmonic function $ReQ(w)$ in the domain, which takes the following continuous values at the boundary:

$$1 < u < u_0^+ : \quad ReQ(u) = \frac{M\gamma(x(u))}{\alpha} \sqrt{\frac{u_0^+ - u}{u-1}};$$

$$u < 0, 0 < u < s_0^-, s_0^+ < u < \infty : \quad ReQ(u) = 0;$$

$$s_0^- < u < 1 : \quad ReQ(u) = -\frac{\Delta P \alpha}{\rho_0 V_0 a} \sqrt{\frac{u_0^+ - u}{1-u}};$$

$$u_0^+ < u < s_0^+ : \quad ReQ(u) = -\frac{\Delta P\alpha}{\rho_0 V_0 a}\sqrt{\frac{u-u_0^+}{u-1}},$$

where $\sin\theta \approx \frac{\gamma}{\alpha}, \alpha = \sqrt{1-M^2}, M = \frac{V_0}{a}$ and $u_0^{\pm}, s_0^{\pm}$ are the roots of algebraic equations

$$l = u_0 - \ln u_0 - 1, \quad s = s_0 - \ln s_0 - 1.$$

The solution for the Dirichlet problem for the semi-plane is given by Schwarz integral:

$$Q(w) = \frac{1}{\pi i}\int_{-\infty}^{\infty} ReQ(t)\frac{dt}{t-w} + iC. \tag{4}$$

In case of a plate the inclination angle is constant and (4) can be taken in elementary functions:

$$Q(w) = -i\frac{\gamma_0 M}{\alpha}\left(\sqrt{\frac{w-u_0^+}{w-1}} - 1\right) +$$

$$i\frac{\Delta P\alpha(u_0^+ + 1)}{\rho_0 V_0 a\pi(u_0^+ - 1)}\ln\left|\frac{\left(1+\sqrt{\frac{s_0^+ - u_0^+}{s_0^+ - 1}}\right)\left(1-\sqrt{\frac{s_0^- - u_0^+}{s_0^- - 1}}\right)}{\left(1-\sqrt{\frac{s_0^+ - u_0^+}{s_0^+ - 1}}\right)\left(1+\sqrt{\frac{s_0^- - u_0^+}{s_0^- - 1}}\right)}\right| +$$

$$+i\frac{\Delta P\alpha(u_0^+ + 1)}{\rho_0 V_0 a\pi(u_0^+ - 1)}\sqrt{\frac{u_0^+ - w}{1-w}}\ln\left|\frac{\left(\sqrt{\frac{u_0^+ - w}{1-w}} + \sqrt{\frac{s_0^- - u_0^+}{s_0^- - 1}}\right)\left(\sqrt{\frac{u_0^+ - w}{1-w}} - \sqrt{\frac{s_0^+ - u_0^+}{s_0^+ - 1}}\right)}{\left(\sqrt{\frac{u_0^+ - w}{1-w}} - \sqrt{\frac{s_0^- - u_0^+}{s_0^- - 1}}\right)\left(\sqrt{\frac{u_0^+ - w}{1-w}} + \sqrt{\frac{s_0^+ - u_0^+}{s_0^+ - 1}}\right)}\right|$$

The obtained solution makes it possible to develop forces affecting on the plate, because pressure distribution in the contact zone is provided by the following expression:

$$p(x(u)) = \frac{M}{\alpha}ReT(u), \quad 1 < u < u_0^+$$

$$p(x(u)) = \left[\frac{M^2\gamma_0}{\alpha} + i\frac{M\Delta P(u_0^+ + 1)}{\rho_0 V_0 a\pi(u_0^+ - 1)}\ln\left|\frac{\left(1+\sqrt{\frac{s_0^+ - u_0^+}{s_0^+ - 1}}\right)\left(1-\sqrt{\frac{s_0^- - u_0^+}{s_0^- - 1}}\right)}{\left(1-\sqrt{\frac{s_0^+ - u_0^+}{s_0^+ - 1}}\right)\left(1+\sqrt{\frac{s_0^- - u_0^+}{s_0^- - 1}}\right)}\right|\right]\sqrt{\frac{u-1}{u_0^+ - u}}.$$

The length of the cavity is determined from the condition of equality of the sum of upper and lower borders of the cavity vertical movements to vertical size of the cavity.

The obtained closed form solution is, however, difficult for being used directly in its current form. In case of relatively small depth the following estimates are valid:

$$l = \frac{\pi L}{\alpha h} \to \infty, \quad s = \frac{\pi S}{\alpha h} \to \infty,$$

$$u_0^+ \approx \frac{\pi L}{\alpha h}, \quad s_0^+ \approx \frac{\pi S}{\alpha h}, \quad s_0^- \approx e^{-\frac{\pi S}{\alpha h}} \approx 0.$$

Then the relation between the lengths of the body and cavity takes the form:

$$\frac{L}{S} = \frac{2e^{-\frac{2\pi L}{\alpha h}}}{1 + e^{-\frac{2\pi L}{\alpha h}} \frac{(\frac{2}{\alpha}-1)\rho_0 V_0^2 \gamma_0 \pi}{\Delta P \alpha} \sqrt{\frac{\pi L}{\alpha h}}}. \tag{5}$$

Using this relation the drag and lift forces in case of cavity formation may be estimated for relatively small depth:

$$\begin{gathered} X = \frac{\rho_0 a^2 M^2 \gamma_0^2 \pi L}{\alpha^3} + \frac{\gamma_0 \Delta P \pi L^2}{2h\alpha^2}(\frac{2\pi L}{\alpha h} + \\ + \ln(1 + e^{-\frac{2\pi L}{\alpha h}} \frac{(\frac{2}{\alpha} - 1)\rho_0 V_0^2 \gamma_0 \pi}{\Delta P \alpha} \sqrt{\frac{\pi L}{\alpha h}})) + \frac{M^2 \gamma_0^2 \Delta P L}{\alpha^3}, \\ Y = -\frac{\rho_0 a^2 M^2 \gamma_0 \pi L}{\alpha^2} - \frac{\Delta P \pi L^2}{2h\alpha}(\frac{2\pi L}{\alpha h} + \\ + \ln(1 + e^{-\frac{2\pi L}{\alpha h}} \frac{(\frac{2}{\alpha} - 1)\rho_0 V_0^2 \gamma_0 \pi}{\Delta P \alpha} \sqrt{\frac{\pi L}{\alpha h}})) - \frac{M^2 \gamma_0 \Delta P L}{\alpha^2}. \end{gathered} \tag{6}$$

Analysis of obtained results shows the behavior of the forces depending on the ratio of body length, fluid layer thickness and pressure in the cavity. As it could be seen from the general solution for resistance and lift forces (6) the forces increase on decreasing pressure in the cavity, which is characterized by increasing ΔP in our problem statement. The limiting values for the forces could be estimated assuming ΔP to be equal to its maximal allowable value $\Delta P = P_0$. On cavity pressure tending to the ambient pressure P_0 ($\Delta P \to 0$) the values of forces coincide exactly with that obtained in [8-10] for an infinite cavity:

$$X = \frac{\rho_0 a^2 M^2 \gamma_0^2 \pi L}{\alpha^3}, \quad Y = -\frac{\rho_0 a^2 M^2 \gamma_0 \pi L}{\alpha^2}.$$

Substituting in (5) pressure differential ΔP being the function of cavitation number one obtains the following formula for the ratio of body and cavity length:

$$\frac{L}{S} = \frac{e^{-\frac{2}{z}}}{1 + e^{-\frac{2}{z}} \frac{2-\alpha}{\alpha^2} \frac{\gamma_0 \pi}{C_{cav.min}} \sqrt{\frac{1}{z}}}, z = \sqrt{\frac{\alpha h}{\pi L}}. \tag{7}$$

It is seen from (7) that the ratio of body to cavity length depends on dimensionless depth z and the cavitation number $C_{cav.min}$. Figure 3 illustrates the dependence of body to cavity length ratio versus dimensionless depth for different values of parameter $A = \frac{2-\alpha}{\alpha^2} \frac{\gamma_0 \pi}{C_{cav.min}}$.

4 Conclusions

The analytical solution was obtained for a problem of body motion in compressible fluid at a final depth with constant velocity and inclination angle. For the case of final length cavity vapor pressure in it is less than the pressure on free surface. The solution allows determining drag and lift forces in the limiting cases. The relation between the body length and the cavity length in the case of small depth or low compressibility was obtained. The forces increase when the pressure in the cavity decreases. When the pressure in the cavity tends to pressure on the free surface the length of the cavity infinitely increase, and forces tend to the values obtained in independent solutions with infinite cavities. The increase of depth under free surface and decrease of compressibility brings to decrease of cavity length.

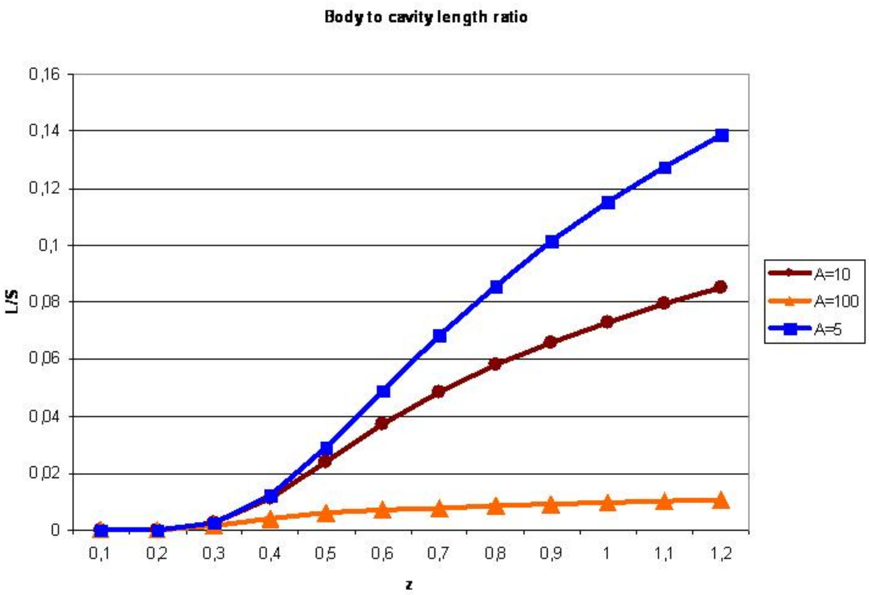

Figure 3: Body to cavity length ratio versus dimensionless depth for different values of parameter A: A=5; 10; 100.

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